The Planck constant and its units

P. R. Bunker, I. M. Mills and Per Jensen

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Mohr P J and Phillips W D*, Metrologia* (2015) 52, 40-47
Mills I, *Metrologia* (2016) 53, 991-997
"A flaw in the SI system," *Nature* (2017) 548, 135
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Bunker P R, Mills I M and Jensen Per, *JQSRT* (2019) 237, 106594(3)

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The Lost Dimension

A flaw in the SI system leaves physicists grappling with ambiguous units.

In SI, angles are considered to be dimensionless and their units are therefore unmentionable.

The mathematical formulation of the laws of nature involves constants. Their values cannot be explained by theory (yet?). They just have to be measured.

There are two types of constant.

With dimensions: Examples are h, c, e, k

and

Dimensionless: Examples are M_p/m_e and α.

The Planck constant

Christians and muslims await the second coming of Jesus

But for physicists there have been many comings of the Planck constant, and each time it involves some sort of quantization. I now list a few important comings.

$E = h\nu$ Planck

$E = h\nu$ Einstein

$p = h/\lambda$ de Broglie

$L = nh/2\pi$ Bohr

$i(h/2\pi)d\Psi/dt = H\Psi$ Quantum Mech.

It does not have the same units at every coming.
But let's first talk about measuring *h*, and the kg.

The measurement of the value of *h*

Phys. Rev., 7, 355 (1916)

Robert Millikan
1868-1953

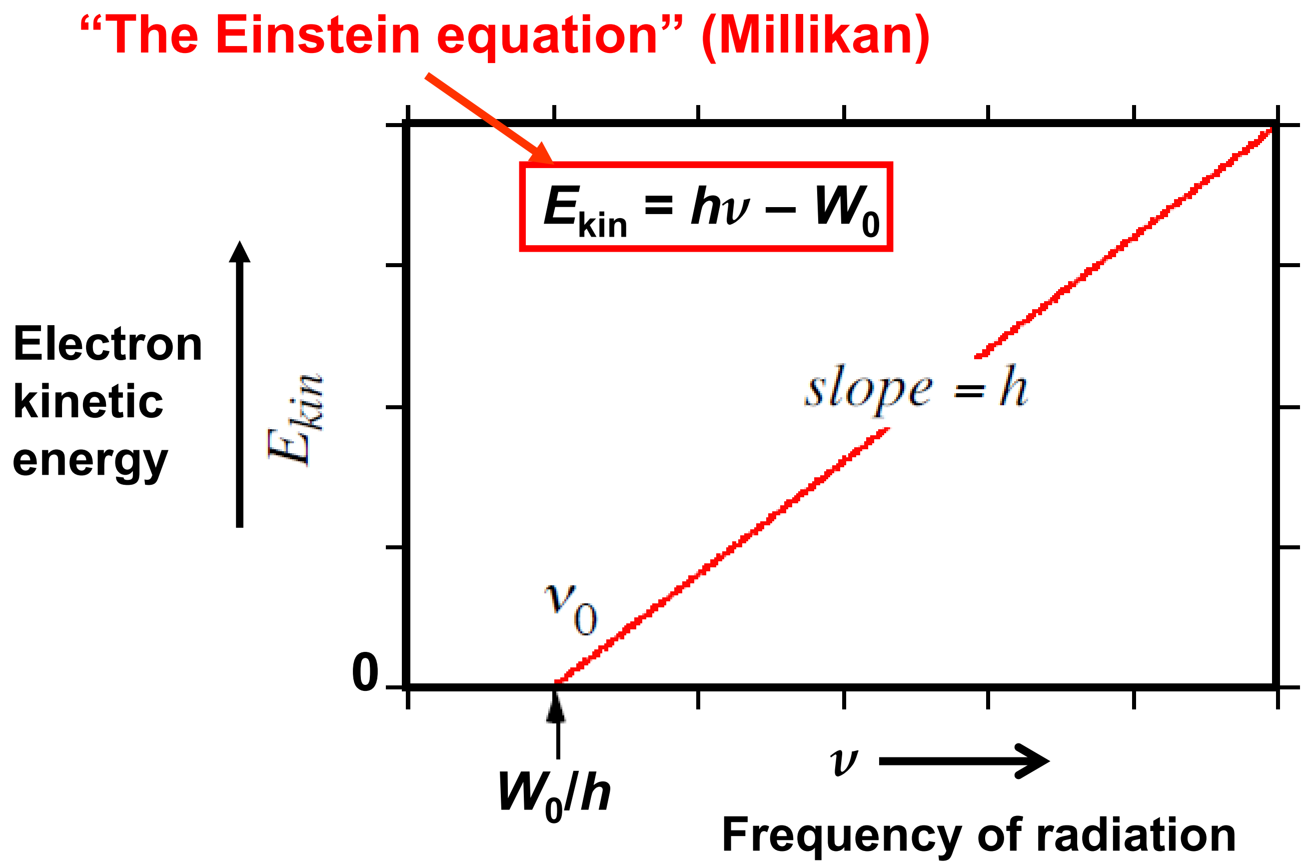
Millikan wanted to test Einstein's theory

$$E(\text{photon}) = h\nu$$

using the photoelectric effect.

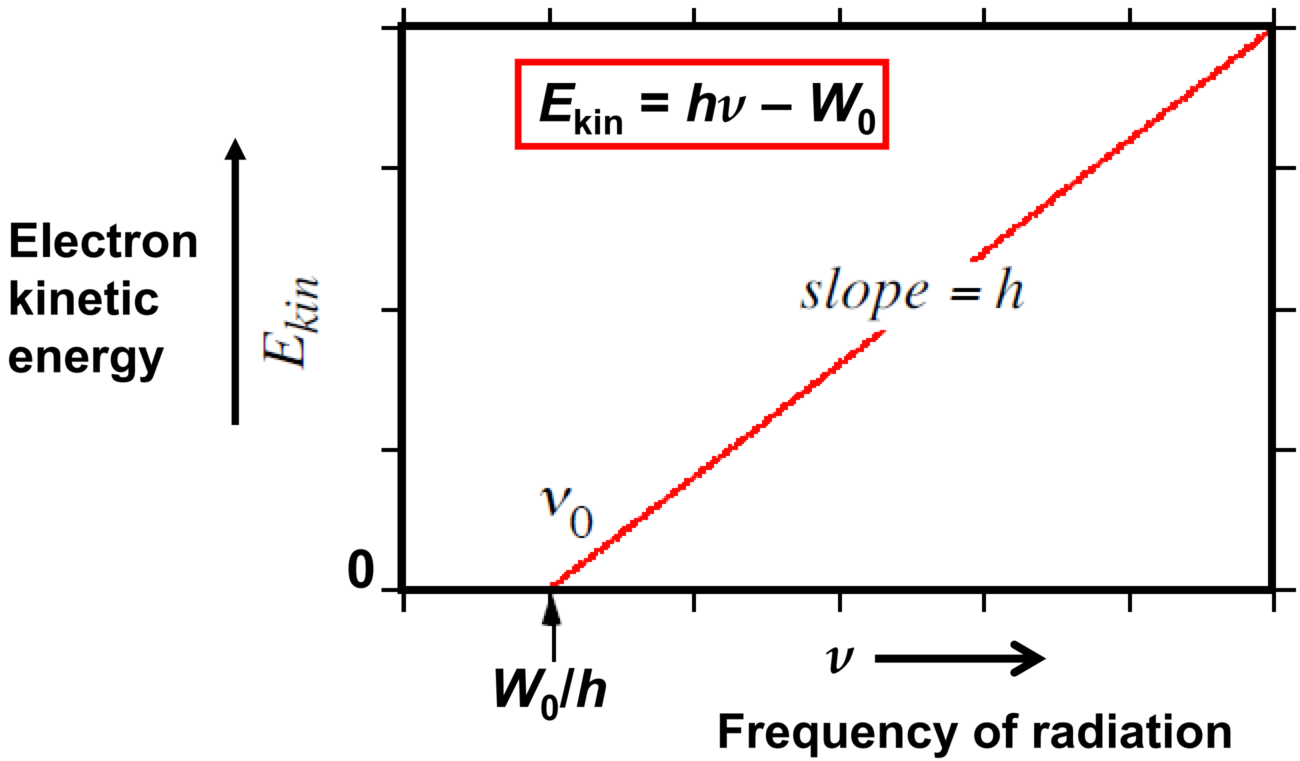
He wanted to prove that Einstein was wrong.

Photoelectron kinetic energy as a function of ν (from $E = h\nu$)



W_0 is the work function: The energy needed to remove an electron.
$W_0 = h\nu_0$

For all materials the slope of E_{kin} vs frequency has slope *h*.



W_0 is the work function: The energy needed to remove an electron.

$W_0 = h\nu_0$

Millikan's paper describes 10 years of work in which he used cleaned lithium and sodium surfaces (in vacuo) with seven different mercury lines (from 2399 Å to 5461 Å) to measure electron energy as a function of frequency.

He confirmed the Einstein equation .

Towards the end of his paper Millikan writes:

Phys. Rev., 7, 355 (1916)

9. THEORIES OF PHOTO EMISSION.

Perhaps it is still too early to assert with absolute confidence the general and exact validity of the Einstein equation. [redacted]

$$E_{\mathrm{kin}} = h\nu - W_0$$

[redacted] the semi-corpuscular theory by which Einstein arrived at his equation seems at present to be wholly untenable.

$E(\mathrm{photon}) = h\nu$

Wave-particle duality

Millikan's grudging summary:

Phys. Rev., 7, 355 (1916)

10. SUMMARY.

1. Einstein's photoelectric equation has been subjected to very searching tests and it appears in every case to predict exactly the observed results.

2. Planck's h has been photoelectrically determined with a precision of about .5 per cent. and is found to have the value

$$h = 6.57 \times 10^{-27}.$$

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO.

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Modern more accurate measurements of the value of *h* have been used to define the 2019 kg.

To help you understand this, I first show how the measurement of the speed of light was used to define the 1983 metre.

QUANTITY CALCULUS (Maxwell)

value of a quantity = numerical value x units

$$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$$

$$c = 29.979\ 245\ 8\ \mathrm{cm\ ns^{-1}}$$

Each of the above is the *value* of the speed of light.

The *numerical values* are:

$$c/(\mathrm{m\ s^{-1}}) = 299\ 792\ 458$$

$$c/(\mathrm{cm\ ns^{-1}}) = 29.979\ 245\ 8$$

1967 – 1983: The value of *c* determined using definitions of 'metre' and 'second'

$$c = 299\ 792\ 458 \text{ m s}^{-1}$$

The value of *c* is fixed by Mother Nature

The numerical value of *c* in m s^{-1} is determined using the definitions of the metre and of the second

Since 1960,
The metre is the length equal to 1 650 763.73 wavelengths of the "krypton 6057 Å line."

Since 1967, the second is the duration of 9 192 631 770 periods of the "9.19 GHz ^{133}Cs hfs transition."

The 1983 definition of the metre

$$c = 299\ 792\ 458\ \text{m s}^{-1}$$

The value of *c* is fixed by Mother Nature

The numerical value of *c* in m s^{-1} is FIXED to be this.

Since 1967, the second is the duration of 9 192 631 770 periods of the "9.19 GHz ^{133}Cs hfs transition."

The metre is the length of the path travelled by light in vacuum in 1/(299 792 458) s.

The numerical value used was determined in a precise spectroscopy experiment so that new metre is consistent with the "Krypton metre."

$c = \lambda\nu$

VOLUME 29, NUMBER 19 PHYSICAL REVIEW LETTERS 6 NOVEMBER 1972

Speed of Light from Direct Frequency and Wavelength Measurements of the Methane-Stabilized Laser

K. M. Evenson, J. S. Wells, F. R. Petersen, B. L. Danielson, and G. W. Day
Quantum Electronics Division, National Bureau of Standards, Boulder, Colorado 80302

and

R. L. Barger* and J. L. Hall†
National Bureau of Standards, Boulder, Colorado 80302
(Received 11 September 1972)

The frequency and wavelength of the methane-stabilized laser at 3.39 μm were directly measured against the respective primary standards. With infrared frequency synthesis techniques, we obtain $\nu = 88.376\,181\,627(50)$ THz. With frequency-controlled interferometry, we find $\lambda = 3.392\,231\,376(12)$ μm. Multiplication yields the speed of light $c = 299\,792\,456.2(1.1)$ m/sec, in agreement with and 100 times less uncertain than the previously accepted value. The main limitation is asymmetry in the krypton 6057-Å line defining the meter.

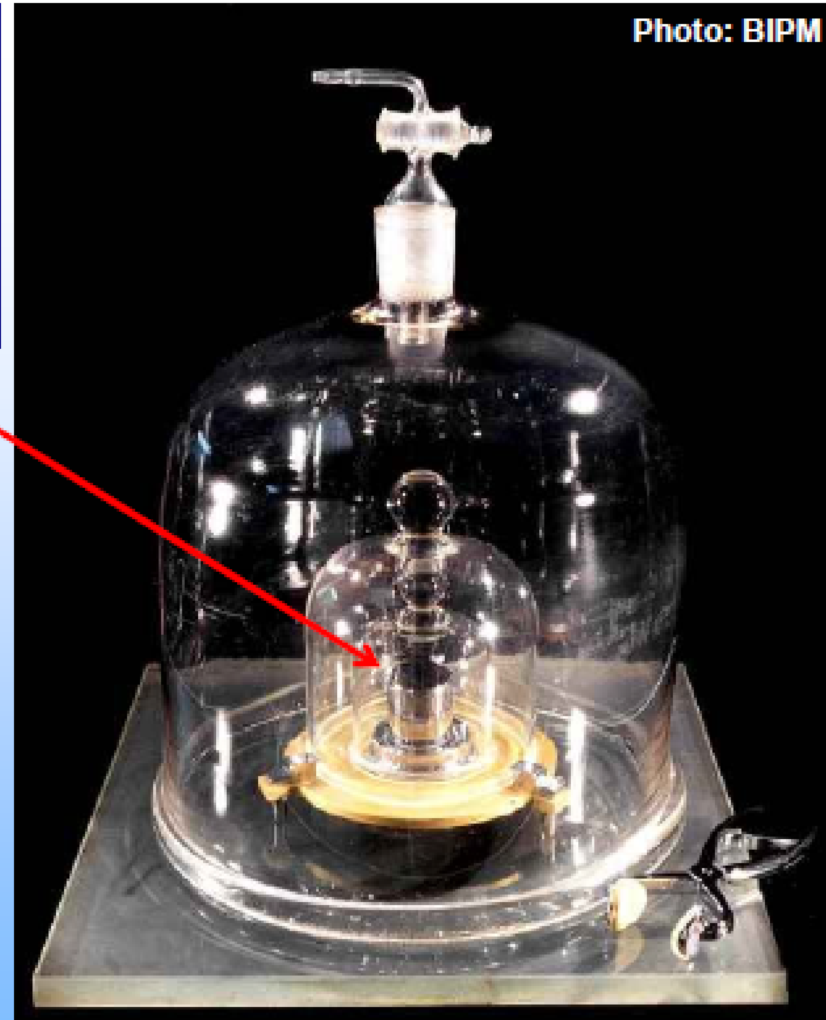
1983 CODATA value 299 792 458 m/s

1889-2019 definition of the kilogram in the SI

The kilogram is the unit of mass;
it is equal to the mass of the international prototype of the kilogram.

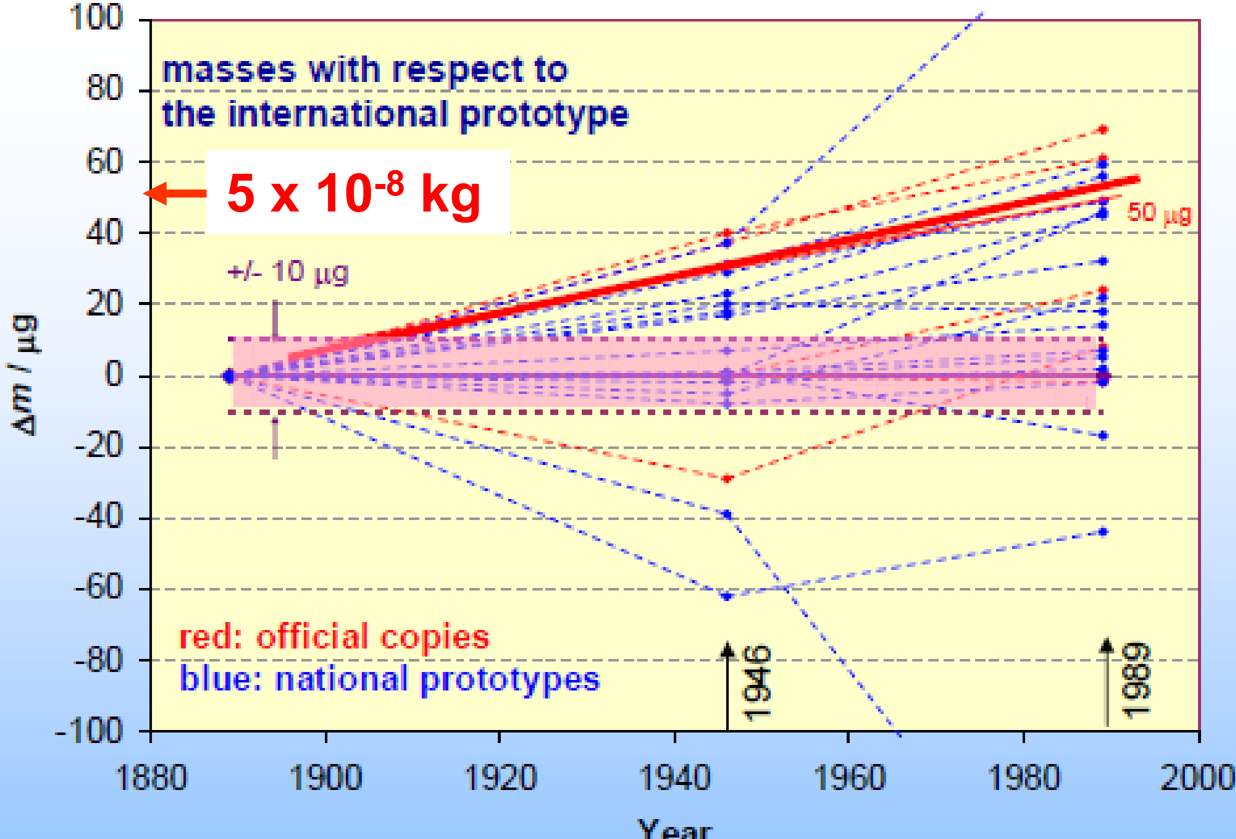
- represents the mass of 1 dm^3 of H_2O at maximum density (4 °C)
- manufactured around 1880, ratified in 1889
- alloy of 90% Pt and 10% Ir
- cylindrical shape, $Ø = h \sim 39$ mm
- kept at the BIPM in ambient air

The kilogram is the last SI base unit defined by a material artefact.



5 official copies made and kept in Paris

Calibration history of the oldest national prototypes



Variations of about **50 µg** (5×10^{-8}) in the mass of the standards **over 100 years**, that is **0.5 µg / year**

Masses of same material can be compared to within 1 µg

A drifing kg also influences the electrical units

Is the IPK losing mass or are the check standards getting heavier ? ?

► Redefinition of the kg in terms of a fundamental constant of nature, for example Planck constant *h* (advantageous for electrical metrology)

The 2019 definition of the kilogram

$$h = 6.626\,070\,15 \times 10^{-34}\ \text{kg m}^2\ \text{s}^{-1}$$

The value of h is fixed by Nature

The numerical value of h in kg m^2 s^{-1} is FIXED to be this.

The second is the duration of 9 192 631 770 periods of the "9.19 GHz ^{133}Cs hfs transition."

The metre is the length of the path travelled by light in vacuum in 1/(299 792 458) s.

The effect of this equation is to define the kg. But first an accurate determination required.

Two highly precise experimental methods for determining the Planck constant

X-ray crystal density

Using a Kibble balance

X-ray crystal density

Determine the number of Silicon atoms (~2x10^{25}) in a highly polished sphere (~94 mm diameter) of pure ^{28}Si with a mass of one kilogram. This gives the Avogadro constant N_A.

$$N_A h = \frac{c A_r(e) M_u \alpha^2}{2R_\infty}$$

M_u = molar mass constant = 10^{-3} kg/mol
A_r(e) = relative atomic mass of electron

[A_r(e) M_u / N_A] = m_e in kg

Achim Leistner at the Australian Centre for Precision Optics (ACPO) holding a one-kilogram single-crystal silicon sphere prepared for the International Avogadro Coordination

The Kibble balance at NRC

Metrologia 54 (2017) 399–409

NRC-17

https://doi.org/10.1088/1681-7575/aa70bf

A summary of the Planck constant determinations using the NRC Kibble balance

B M Wood, C A Sanchez, R G Green and J O Liard

National Research Council of Canada, 1200 Montreal Road, Ottawa, ON K1A 0R6, Canada

E-mail: barry.wood@nrc-cnrc.gc.ca

Received 31 March 2017, revised 28 April 2017
Accepted for publication 3 May 2017
Published 30 May 2017

CrossMark

The last determination of the numerical value of the Planck constant.

After fixing the numerical value of *h*, the Kibble balance will become an instrument for measuring absolute mass

We could then determine the mass of each kilogram artifact on a regular basis.

h is not the only dimensioned constant that gets a defined numerical value in 2019

Newell D B et al., *Metrologia* **(2018) 55** L13.

Quantity	Value	Redefined base unit
h	$6.626\,070\,15 \times 10^{-34}$ J s	kg
e	$1.602\,176\,634 \times 10^{-19}$ C	ampere
k	$1.380\,649 \times 10^{-23}$ J K^{-1}	K
N_A	$6.022\,140\,76 \times 10^{23}$ mol^{-1}	mole

See www.bipm.org

C = A s

J = kg m^2 s^{-2}

The base units in the International System of Units (SI).

h is not the only dimensioned constant that gets a defined numerical value in 2019

Newell D B et al., *Metrologia* **(2018) 55** L13.

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See www.bipm.org

Now to looking at this

J s are units of action

J s = kg m^2 s^{-1}

QUANTITY CALCULUS (Maxwell)

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$$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$$

$$c = 29.979\ 245\ 8\ \mathrm{cm\ ns^{-1}}$$

CORRECT BUT NOT QUANTITY CALCULUS:

$$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$$

$$d = 29.979\ 245\ 8\ \mathrm{cm\ ns^{-1}}$$

d=c = the value of the speed of light

$$d/(\mathrm{cm\ ns^{-1}}) = [c/(\mathrm{m\ s^{-1}})]/10^7$$

QUANTITY CALCULUS (Maxwell)

value of a quantity = numerical value x units

$$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$$

$$c = 29.979\ 245\ 8\ \mathrm{cm\ ns^{-1}}$$

CORRECT BUT NOT QUANTITY CALCULUS:

$$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$$

$$d = 29.979\ 245\ 8\ \mathrm{cm\ ns^{-1}}$$

***d*=*c* = the value of the speed of light**

$$d/(\mathrm{cm\ ns^{-1}}) = [c/(\mathrm{m\ s^{-1}})]/10^7$$

WRONG is $d = \mathrm{c}/10^7 = 29.979\ 245\ 8\ \mathrm{m\ s^{-1}}$

The units used for frequency

Suppose we have a frequency of 100 cycle s^{-1}

ν = 100 cycle s^{-1}

= 2π x 100 radian s^{-1}

The units used for frequency

Suppose we have a frequency of 100 cycle s^{-1}

ν = 100 cycle s^{-1}

With the units radian s^{-1} spectroscopists use ω

$\omega = 2\pi \times 100$ radian s^{-1}

$\omega = \nu$ but ratio of numerical values is 2π

The units used for frequency

Suppose we have a frequency of 100 cycle s^{-1}

ν = 100 cycle s^{-1}

With the units radian s^{-1} spectroscopists use ω

$\omega = 2\pi$ x 100 radian s^{-1}

$\omega = \nu$ but ratio of numerical values is 2π

omitting the units of angular measure

ν = 100 s^{-1}

$\omega = 2\pi$ x 100 s^{-1}

and it is said that $\omega = 2\pi\nu$.

In fact $\omega = \nu$ and their numerical values have the ratio 2π.

In SI we replace s^{-1} with Hz to further obscure the real units

A PONTIFICATION

Always explicitly state whether a frequency is in cycle s^{-1} or radian s^{-1}. Use Hz for cycle s^{-1}.

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Always explicitly state whether a frequency is in cycle s^{-1} or radian s^{-1}. Use Hz for cycle s^{-1}.

Now finally to the units of the Planck constant

The units of the Planck constant

We know that

$$h = m_e c \alpha^2/(2R_\infty)$$

The units of wavenumber

Units: m_e in kg, c in m s^{-1}, and R_∞ in cycle m^{-1}
gives h the units kg m^2 s^{-1} $cycle^{-1}$ = J s $cycle^{-1}$
= J/(cycle s^{-1})
= J/Hz.

h = 6.626 070 150 x 10^{-34} J s $cycle^{-1}$

The units of the Planck constant

We know that

This *h* has the dimensions of action/angle, energy/frequency and of angular momentum

of
wavenumber

Units: m_e in kg, c in m s^{-1}, and R_∞ in cycle m^{-1}
gives h the units kg m^2 s^{-1} cycle^{-1} = J s cycle^{-1}
= J/(cycle s^{-1})
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h = 6.626 070 150 x 10^{-34} J s cycle^{-1}

The units of *h* and *ħ* from paper by PRB, Mills and Jensen.

Using E = *hν*, *h* is Energy/Frequency

Energy: Units are J = kg m^2 s^{-2}
Frequency: Units are radian s^{-1} or cycle s^{-1}

If we measure frequency in cycle s^{-1} = Hz
the units for *h* are

J/(cycle s^{-1}) = J/Hz = J s cycle^{-1}. ← **Just as obtained from** $h = m_e c\alpha^2/(2R_\infty)$

If we measure frequency in radians s^{-1}
the units for *h* are

J/(radian s^{-1}) = J s radian^{-1}.

Applying rules of quantity calculus:

$$h = 6.626\ 070\ 150 \times 10^{-34}\ \text{J s cycle}^{-1}$$

$$h = 1.054\ 571\ 818\ldots \times 10^{-34}\ \text{J s radian}^{-1}$$

$$\boxed{\text{Numerical value of } h \text{ in J s radian}^{-1}} = \left[\boxed{\text{Numerical value of } h \text{ in J s cycle}^{-1}} / 2\pi\right]$$

Note that units are those of action/angle

Within SI:

h = 6.626 070 150 x 10^{-34} J s,

$\hbar$ = 1.054 571 818…x 10^{-34} J s,
they are given the same units, and
it is said that $\hbar = h/2\pi$.

In fact (angular units included) $h = \hbar$. *Each* is the value of the PC but with different units. Their numerical values have the ratio 2π.

Correct (but not quantity calculus) is:

h = 6.626 070 150 x 10^{-34} J s cycle^{-1}

$\hbar$ = 1.054 571 818… x 10^{-34} J s radian^{-1}

We will call $\hbar$ the reduced Planck constant.

N.B. $h = \hbar$, but they have different units.
Their numerical values have the ratio 2π.

We do not have $\hbar = h/2\pi$

Table 4. Fundamental constants and their units.

Constants	Symbol	Units
Reduced Planck constant	$\hbar$	$\mathrm{J\ s\ rad^{-1}}$
Planck constant	h	$\mathrm{J\ Hz^{-1}}$
Electron reduced Compton wavelength	$\lambda\!\!\!^{-}_{C}$	$\mathrm{m\ rad^{-1}}$
Electron Bohr radius	a_0	$\mathrm{m\ rad^{-1}}$
Rydberg constant	$R\!\!\!-_{\infty}$	$\mathrm{rad\ m^{-1}}$
Vacuum permittivity (electric constant)	ϵ_0	$\mathrm{C^2\ J^{-1}\ rad\ m^{-1}}$
Vacuum permeability (magnetic constant)	μ_0	$\mathrm{kg\ m\ rad^{-1}}\ C^{-2}$
Avogadro constant	N_A	$\mathrm{ent\ mol^{-1}}$
Boltzmann constant	k	$\mathrm{J\ K^{-1}\ ent^{-1}}$
Elementary charge	e	$\mathrm{C\ ent^{-1}}$

Summary of the units and dimensions of *h* and *ħ*

$E = h\nu$ Planck Einstein

h is energy/frequency
J/(cycle/s) = J s cycle^{-1}

The Planck constant: h = 6.626 070 150 x 10^{-34} J s cycle^{-1}

$p = h/\lambda$ de Broglie

h is momentum x wavelength
(kg m s^{-1})(m cycle^{-1})= J s cycle^{-1}

$L = n\text{“}h/2\pi\text{”} = n\hbar$ Bohr

The reduced PC: $\hbar$ = 1.054 571 818… x 10^{-34} J s radian^{-1}

$h = \hbar$ but they are in different units and the numerical values are in the ratio of 2π

Summary of the units and dimensions of h and $\hbar$

$E = h\nu$ Planck Einstein

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The reduced PC: $\hbar$ = 1.054 571 818… x 10^{-34} J s radian^{-1}

BUT Quantum Mechanics gives rise to the coming of another variety of the Planck constant

Dirac's introduces $\hbar$ as "a new universal constant" having the dimensions of action.

Pp 86-87 from Dirac's Book "The Principles of Quantum Mechanics" (1930)

From analogy between quantum mechanics and classical mechanics

"Let us try to introduce a quantum Poisson Bracket which shall be the analogue of the classical one………"

This leads to an expression involving a commutator and a real constant factor that Dirac calls $\hbar$, and after this expression he says:

in which $\hbar$ is a new universal constant. It has the dimensions of action. In order that the theory may agree with experiment, we must take $\hbar$ equal to $h/2\pi$, where h is the universal constant that was introduced by Planck, known as Planck's constant.

Action = energy times time, units J s

But *h* has dimensions of action/angle units J s cycle^{-1}

Units and dimensions

$E = h\nu$ **Planck Einstein**

***h* is energy/frequency**
J/(cycle/s) = J s cycle^{-1}

The Planck constant: h = 6.626 070 150 x 10^{-34} J s cycle^{-1}

$p = h/\lambda$ **de Broglie**

***h* is momentum x wavelength**
(kg m s^{-1})(m cycle^{-1})= J s cycle^{-1}

$L = n$"$h/2\pi$" $= n\hbar$ **Bohr**

The reduced PC: $\hbar$ = 1.054 571 818… x 10^{-34} J s radian^{-1}

Heisenberg, Schrödinger and Dirac

$i(h/2\pi)d\Psi/dt = \hat{H}\Psi$ $\qquad$ $-i(h/2\pi)d\Psi/dq = \hat{p}\Psi$

J s	s^{-1}	J	J s = kg m^2 s^{-1}	m^{-1}	kg m s^{-1}

Units and dimensions

$E = h\nu$ Planck Einstein

h is energy/frequency
J/(cycle/s) = J s cycle^{-1}

The Planck constant: $h = 6.626\ 070\ 150 \times 10^{-34}$ J s cycle^{-1}

$p = h/\lambda$ de Broglie

h is momentum x wavelength
(kg m s^{-1})(m cycle^{-1})= J s cycle^{-1}

$L = n$"$h/2\pi$" $= n\hbar$ Bohr

The reduced PC: $\hbar = 1.054\ 571\ 818\ldots \times 10^{-34}$ J s radian^{-1}

Heisenberg, Schrödinger and Dirac

$i(h_A/2\pi)d\Psi/dt = \hat{H}\Psi$ $-i(h_A/2\pi)d\Psi/dq = \hat{p}\Psi$

The Planck constant of action: $h_A = 6.626\ 070\ 150 \times 10^{-34}$ J s

There are two h and two $\hbar$

1. Planck, Einstein and de Broglie. $E = h\nu$, and $p = h/\lambda$.

The Planck constant: $h = 6.626\ 070\ 150 \times 10^{-34}$ J s cycle^{-1}

2. Bohr. Angular momentum quantization; called $\hbar$.

The reduced Planck constant:

$$\hbar = 1.054\ 571\ 818\ldots \times 10^{-34} \text{ J s radian}^{-1}$$

The numerical values of h and $\hbar$ are in the ratio 2π.

3. von Klitzing. The von Klitzing constant $R_K = h_A/e^2$.

The Planck constant of action:

$$h_A = 6.626\ 070\ 150 \times 10^{-34} \text{ J s}$$

4. Heisenberg, Schrödinger and Dirac; this is $h_A/2\pi$. We call it $\hbar_A$.

The reduced Planck constant of action:

$$\hbar_A = 1.054\ 571\ 818\ldots \times 10^{-34} \text{ J s}$$

The reduced Planck constant of action is what is called "The Dirac constant" in the current literature.

Kibble balance equations

Measure voltage U_1. Use the Josephson effect

$$U_1 = h\nu_1/(2e). \quad U_1 \text{ to } \nu_1 \text{ conversion.}$$

The voltage across the barrier layer is the product of $h/(2e)$ and the time derivative of the phase difference ν_1 (in cycle s^{-1}). h in J s $cycle^{-1}$.

The units of $U_1 = h\nu_1/(2e)$ are (J s $cycle^{-1}$)(cycle s^{-1})/(A s) = J/(A s) = volt.

Measure current I. Use Josephson and quantum Hall effects

$$I = U_2/R_K = [h\nu_2/(2e)] / R_K$$

The von Klitzing constant $R_K = h_A/e^2$. The constant h_A in J s.

The units of h_A/e^2 are (J s)/(A^2 s^2) = J s^{-1} A^{-2} = ohm.

So $\quad I = U_2/R_K = [h\nu_2/(2e)] / (h_A/e^2) = (h/h_A)(\nu_2 e/2)$

Note that $(h/h_A) = 1$ $cycle^{-1}$

The units of $I = (h/h_A)(\nu_2 e/2)$ are ($cycle^{-1}$)(cycle s^{-1})(A s) = A, ampere.

From this we get $\quad U_1 I = h(h/h_A)\nu_1\nu_2/4$ with units J s^{-1}

and thus $h = 4U_1 I(h_A/h)/(\nu_1\nu_2)$ with units J s $cycle^{-1}$.

where $h_A/h = 1$ cycle

Getting the units of *h* from its relation to the Avogadro constant.

$$N_A h = \frac{c A_r(e) M_u \alpha^2}{2R_\infty}$$

N_A	Avogadro-constant
h	*Planck-constant*
c	*speed of light*
α	*fine-structure constant*
R_∞	*Rydberg constant*
e	*elementary charge*
M_u	*molar mass constant = 1x10⁻³ kg/mol*

A_r(e) and α are dimensionless.

M_u/N_A has units of kg.

R_∞ has units of cycle m^{-1}

Units of h are those of $c(M_u/N_A)/R_\infty$,

which is (m/s) kg m cycle^{-1} = J s/cycle

X-ray crystal density

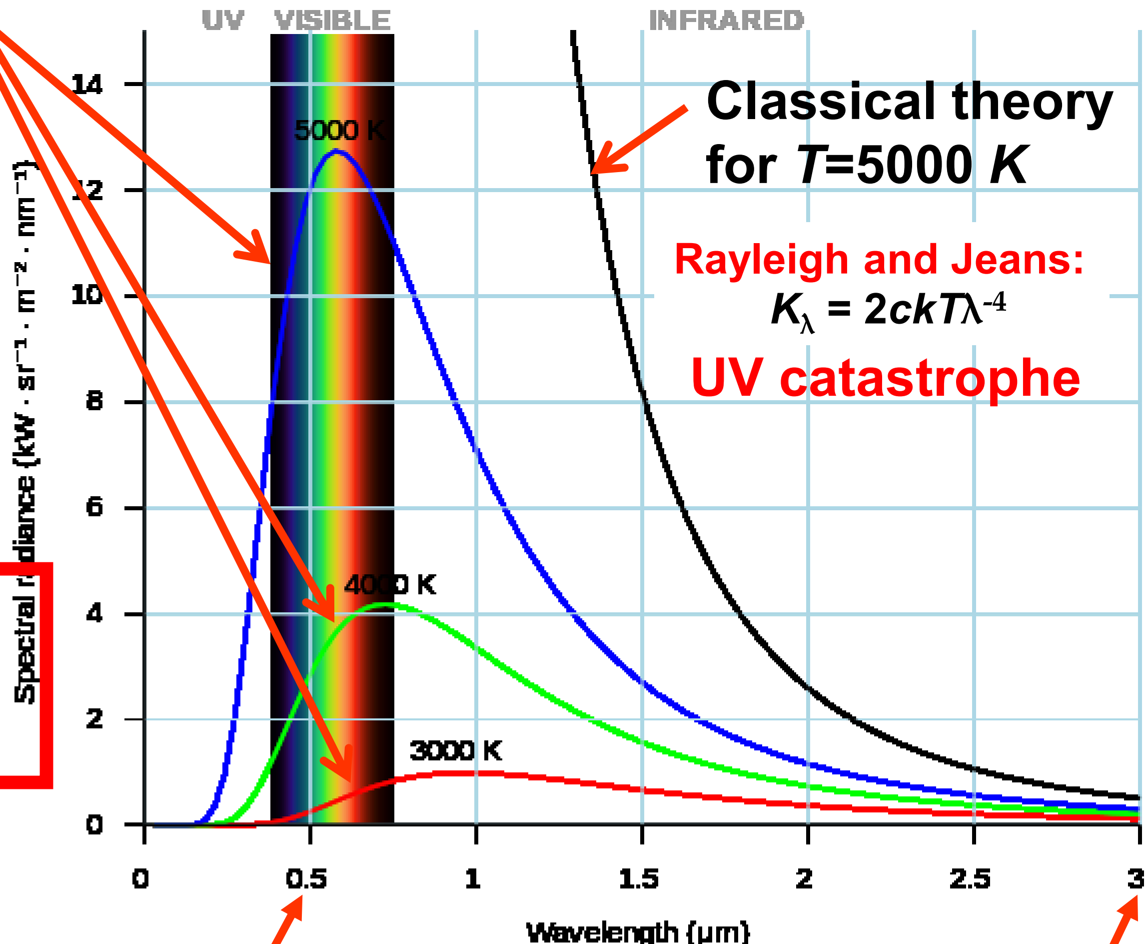
Spectral Distribution Function $I(\lambda, T) = K_\lambda$ of Black-Body Radiation. The Wien expression.

experiment

First experiment
Langley 1886

More experiments
Paschen and Wien
1896-1899

Wien gets empirical expression:
$K_\lambda = b\lambda^{-5}/[exp(a/\lambda T)]$



Classical theory for T=5000 K

Rayleigh and Jeans:
$K_\lambda = 2ckT\lambda^{-4}$

UV catastrophe

5000 Å
20000 cm^{-1}

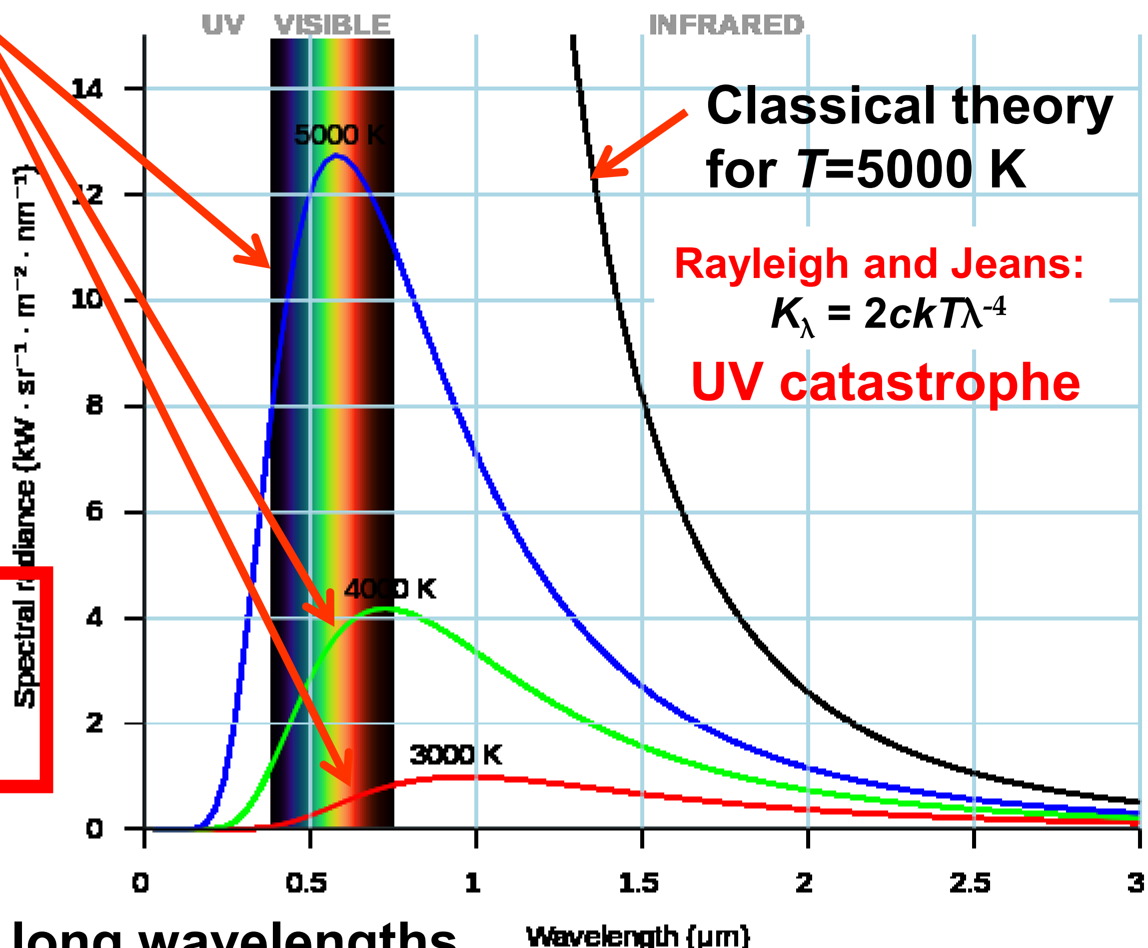
3333 cm^{-1}

Spectral Distribution Function $I(\lambda, T) = K_\lambda$ of Black-Body Radiation. Wien is no good at large λ.

experiment

First experiment
Langley 1886

More experiments
Paschen and Wien
1896-1899



Classical theory
for T=5000 K

Rayleigh and Jeans:
$K_\lambda = 2ckT\lambda^{-4}$

UV catastrophe

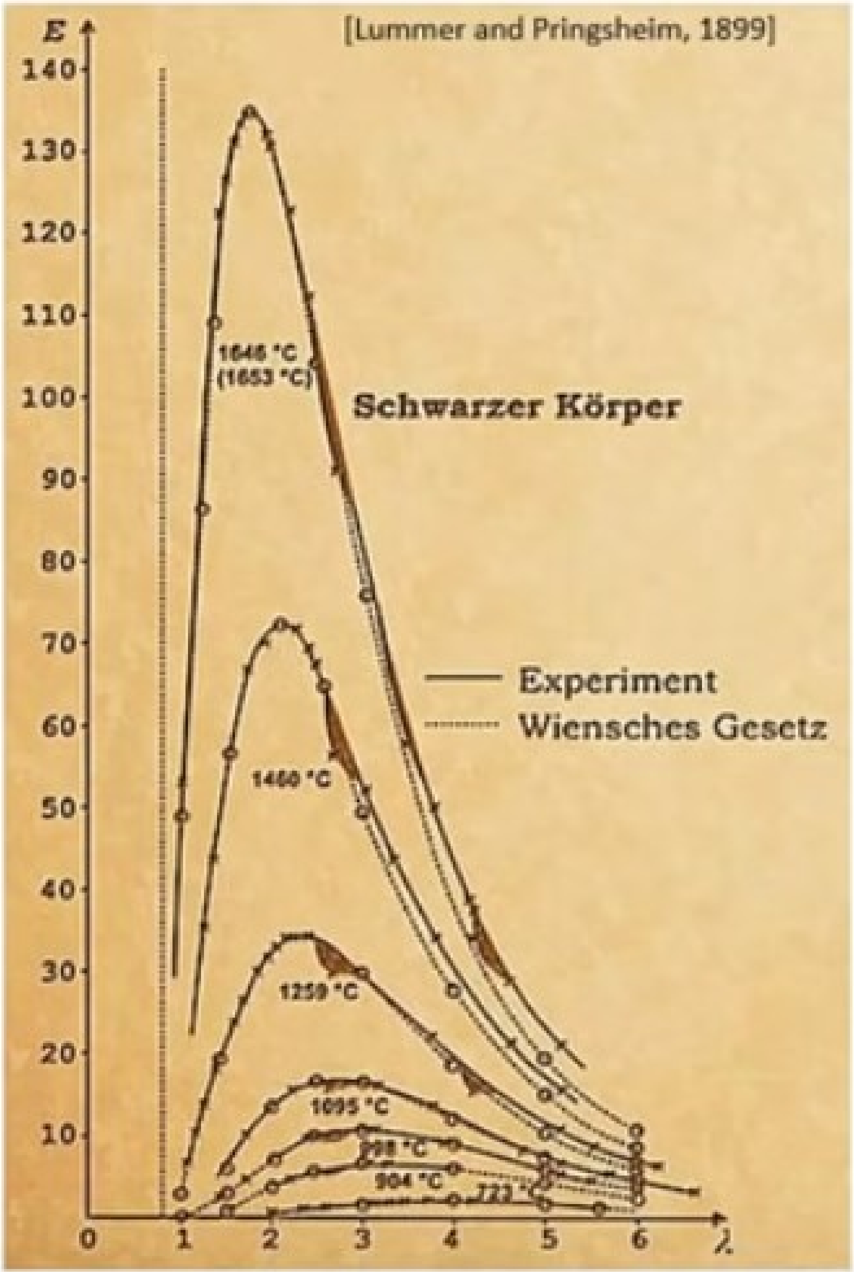
Wien gets empirical expression:
$K_\lambda = b\lambda^{-5}/[\exp(a/\lambda T)]$

Wien K_λ too small at long wavelengths

At large λ fits $K_\lambda = 2ck\mathrm{T}\lambda^{-4}$

1899 Lummer, Pringsheim, Rubens, and Kurlbaum experiments to 50 μm

Max Planck (1900): Fits Black Body Radiation



AN EMPIRICAL FIT

IV.

Verhandlungen der physikal. Gesellschaft 2, p. 202; 1900.

Ueber eine Verbesserung der Wienschen Spektralgleichung. (1)

(Vorgetragen in der Sitzung vom 19. Oktober 1900.)

$$e_\lambda = \frac{C\lambda^{-5}}{e^{c'/\lambda T} - 1}$$

Planck 'adds' this "-1" to the denominator of Wien's expression. Fits at high λ.

14 December

Max Planck(1900): DERIVES his Distribution Law by assuming Black Body "oscillators" are quantized

"Oscillators" in the BB are quantized

$E = h\nu$

The birth of the Planck constant

Leads to

Planck's Distribution Law

$e_\lambda = C\lambda^{-5} / [\exp(c'/\lambda T) - 1]$

$C = 2hc^2$

$c'/\lambda T = h\nu/kT$

At large λ: $\exp(c'/\lambda T) = 1 + (c'/\lambda T)$, and we get

Rayleigh-Jeans classical expression

$e_\lambda = 2ckT\lambda^{-4}$

Planck constant(s): birth

9. Ueber das Gesetz der Energieverteilung im Normalspectrum; von Max Planck.

(In anderer Form mitgeteilt in der Deutschen Physikalischen Gesellschaft, Sitzung vom 19. October und vom 14. December 1900, Verhandlungen 2. p. 202 und p. 237. 1900.)

§ 2. Wir setzen nun die Entropie S_N des Systems, bis auf eine willkürlich bleibende additive Constante, proportional dem Logarithmus der Wahrscheinlichkeit W dafür, dass die N Resonatoren insgesamt die Energie U_N besitzen, also:

(3) $$S_N = k \log W + \text{const.}$$

§ 10. Wenden wir das Wien'sche Verschiebungsgesetz in der letzten Fassung auf den Ausdruck (6) der Entropie S an, so erkennen wir, dass das Energieelement ε proportional der Schwingungszahl ν sein muss, also:

$$\varepsilon = h \cdot \nu$$

Unnumbered equation

Hierbei sind h und k universelle Constante.

Hieraus und aus (14) ergeben sich die Werte der Naturconstanten:

(15) $$h = 6{,}55 \cdot 10^{-27}\ \text{erg} \cdot \text{sec},$$

(16) $$k = 1{,}346 \cdot 10^{-16}\ \frac{\text{erg}}{\text{grad}}.$$

Einstein (1905): EM radiation is Quantized

Einstein's famous paper with the title:

"On a heuristic viewpoint concerning the production and transformation of light."

Introduces photons with energy $E = h\nu$

"It endowed Planck's quantum hypothesis with physical reality. The oscillators for which Planck proposed energy quantization were fictitious, and his theory for blackbody radiation lacked obvious physical consequences. But the radiation theory for which Einstein proposed energy quantization was real, and his theory had immediate physical consequences."
D. Kleppner, Physics Today, February 2005 page 30.