The Planck constant, its units and the kg.

Number of ppt images is 32

QUANTITY CALCULUS (Maxwell)

value of a quantity = numerical value x units

$$c = 299\ 792\ 458 \text{ m s}^{-1}$$
$$c = 29.979\ 245\ 8 \text{ cm ns}^{-1}$$

$$c/(\text{m s}^{-1}) = 299\ 792\ 458$$
$$c/(\text{cm ns}^{-1}) = 29.979\ 245\ 8$$

QUANTITY CALCULUS (Maxwell)

value of a quantity = numerical value x units

$$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$$

$$c = 29.979\ 245\ 8\ \mathrm{cm\ ns^{-1}}$$

NOT QUANTITY CALCULUS

$$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$$

$$\boxed{d = 29.979\ 245\ 8\ \mathrm{cm\ ns^{-1}}}$$

$c=d$ = the value of the speed of light

$$\boxed{\text{Numerical value of } d} = 10^{-7} \times \boxed{\text{Numerical value of } c}$$

The units we use for frequency

Suppose $\nu = 100$ cycle s^{-1} $= 2\pi \times 100$ radian s^{-1}.

In SI we write $\omega = 2\pi \times 100$ radian s^{-1}

$\nu = \omega$ but ratio of numerical values is 2π

In SI we omit the units of angular measure:

$\nu = 100\ s^{-1}$

$\omega = 2\pi \times 100\ s^{-1}$

SI obscures things further by saying s^{-1} = Hz:

$\nu = 100$ Hz

$\omega = 2\pi \times 100$ Hz

Spectroscopists should always explicitly state whether a frequency is in cycle s^{-1} or radian s^{-1}. Also the Hz should be defined as cycle s^{-1}.

Now to the Planck constant

h arises in Physics when there is quantization

Planck (black bodies) $E = h\nu$

Einstein (photons) $E = h\nu$

Light has particle-wave duality

De Broglie (particles) $E = h\nu$, $p = h/\lambda$

Bohr (H-atom):

Spectral lines have $h\nu = (E_1 - E_2)$

Stationary states have quantized angular momentum $= nh/2\pi$.

Quantum mechanics:

Heisenberg, *Zeits. f. Phys.*, 33, 879-893 (1925)

For the harmonic oscillator
Heisenberg gets:

Note ω and $h/2\pi$

$$W = (n + \tfrac{1}{2}) h\omega_0/2\pi \qquad (23)$$

Schrödinger, *Ann. D. Phys.*, 79, 361-376 (1926)

Schrödinger wave equation has $K = h/2\pi$

Dirac, *Proc. Roy. Soc.*, A112, 661-677 (1926)

Dirac writes: "where $\hbar$ is $(2\pi)^{-1}$ times the usual Planck's constant"

Introduction of $\hbar$ notation. Dirac visited Malta?

Dirac's introduction of $\hbar$

Pp 86-87 from Dirac's Book "The Principles of Quantum Mechanics" (1930)

Dirac develops an equation that involves the quantum mechanical equivalent of a classical Poisson bracket. This equation involves a real constant for which he uses the symbol $\hbar$.

He then says:

$\hbar$ is a new universal constant. It has the dimensions of action. In order that the theory may agree with experiment, we must take $\hbar$ equal to $h/2\pi$, where h is the universal constant that was introduced by Planck, known as Planck's constant.

Now to the units of the Planck constant

1960 – 1983: The definition of the speed of light using the metre and the second.

$$c = 299\ 792\ 458 \text{ m s}^{-1}$$

The value of *c* is fixed by Mother Nature

The numerical value of *c* in m s^{-1} is determined using the definitions of the metre and of the second

The metre is the length equal to 1 650 763.73 wavelengths of the "krypton 6057 Å line."

The second is the duration of 9 192 631 770 periods of the "9.19 GHz ^{133}Cs hfs transition."

The post-1983 definition of the metre

$$c = 299\ 792\ 458\ \text{m s}^{-1}$$

The value of *c* is fixed by Mother Nature

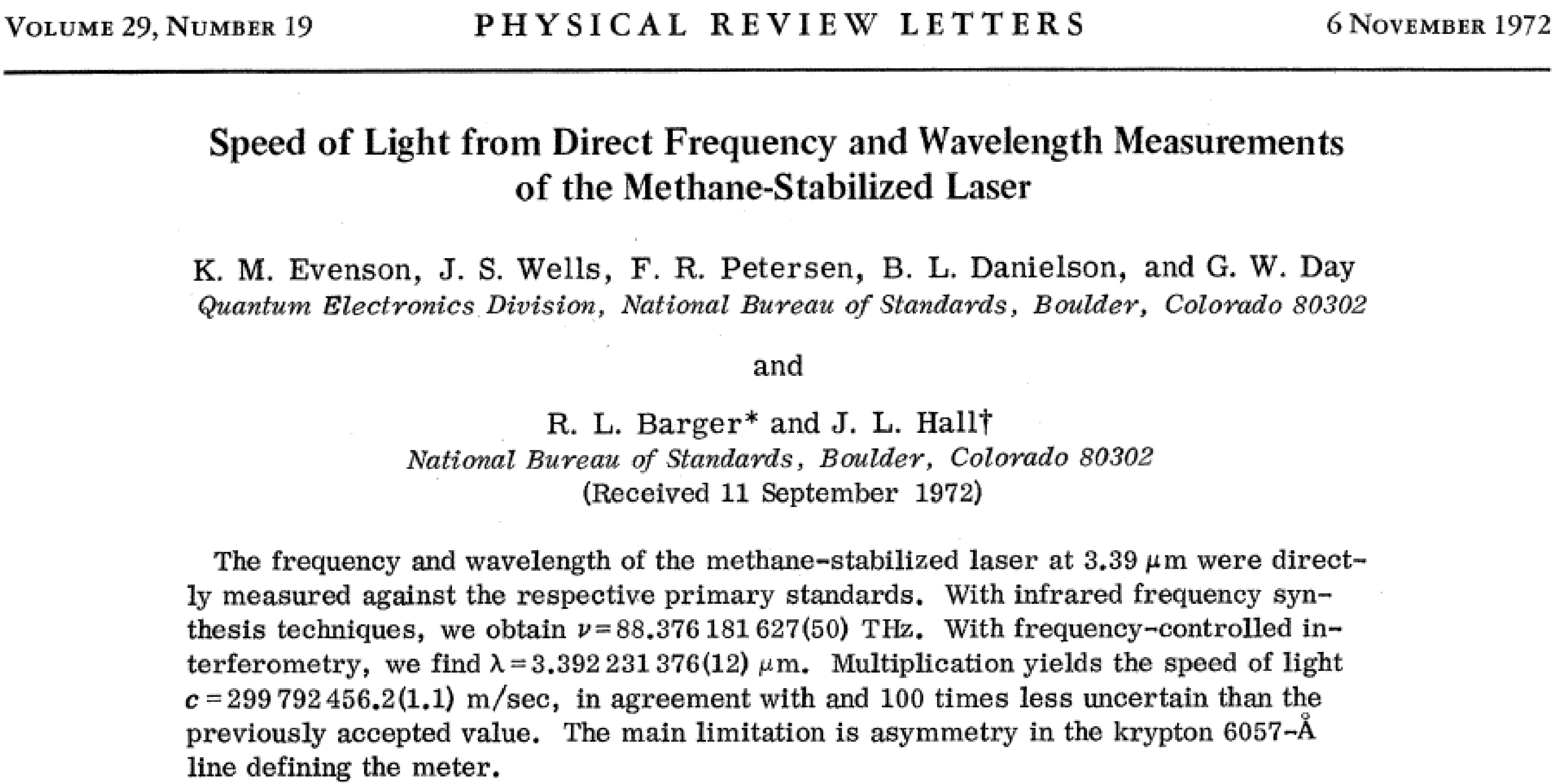
The numerical value of *c* in m s^{-1} is now FIXED to be this.

The second is the duration of 9 192 631 770 periods of the "9.19 GHz ^{133}Cs hfs transition."

The metre is the length of the path travelled by light in vacuum in 1/(299 792 458) s.

The numerical value used was determined in a precise spectroscopy experiment so that new metre is consistent with the "Krypton metre."

$c = \lambda\nu$


VOLUME 29, NUMBER 19 PHYSICAL REVIEW LETTERS 6 NOVEMBER 1972

Speed of Light from Direct Frequency and Wavelength Measurements of the Methane-Stabilized Laser

K. M. Evenson, J. S. Wells, F. R. Petersen, B. L. Danielson, and G. W. Day
Quantum Electronics Division, National Bureau of Standards, Boulder, Colorado 80302

and

R. L. Barger* and J. L. Hall†
National Bureau of Standards, Boulder, Colorado 80302
(Received 11 September 1972)

The frequency and wavelength of the methane-stabilized laser at 3.39 μm were directly measured against the respective primary standards. With infrared frequency synthesis techniques, we obtain $\nu = 88.376\,181\,627(50)$ THz. With frequency-controlled interferometry, we find $\lambda = 3.392\,231\,376(12)$ μm. Multiplication yields the speed of light $c = 299\,792\,456.2(1.1)$ m/sec, in agreement with and 100 times less uncertain than the previously accepted value. The main limitation is asymmetry in the krypton 6057-Å line defining the meter.

CODATA least squares fitted value is 299 792 458 m s^{-1}

The 2019 definition of the kilogram

$$h = 6.626\ 070\ 15 \times 10^{-34}\ \text{kg}\ \text{m}^2\ \text{s}^{-1}$$

The value of h is
fixed by Nature

The numerical value of h
in kg m^2 s^{-1} is FIXED to be this.

The second is the duration of 9 192 631 770
periods of the "9.19 GHz ^{133}Cs hfs transition."

The metre is the length of the path travelled by light in vacuum in 1/(299 792 458) s.

The effect of this equation is to define the kg.

The 2019 definition of the kilogram

$$h = 6.626\ 070\ 15 \times 10^{-34}\ \boxed{\text{kg m}^2\ \text{s}^{-1}}$$

The value of h is
fixed by Nature

Th
in

BUT HOW DO WE GET THESE UNITS?

The metre is the length of the path travelled by light in vacuum in 1/(299 792 458) s.

The effect of this equation is to define the kg.

The units of *h*

$E = h\nu$, so h is Energy/Frequency

Energy: Units are $J = kg\ m^2\ s^{-2}$
Frequency: Units are radian s^{-1} or cycle s^{-1}

Thus, if we measure frequency in radians s^{-1}
the units for h are

$$J/(\text{radian } s^{-1}) = J\ s\ \text{radian}^{-1} = kg\ m^2\ s^{-1}\ \text{radian}^{-1}.$$

And, if we measure frequency in cycle s^{-1}
the units for h are

$$J/(\text{cycle } s^{-1}) = J\ s\ \text{cycle}^{-1} = kg\ m^2\ s^{-1}\ \boxed{\text{cycle}^{-1}.}$$

The 2019 SI definition of the kg

$$h = 6.626\ 070\ 15 \times 10^{-34}\ kg\ m^2\ s^{-1}$$

The correct use of quantity calculus:

$$h = 6.626\ 070\ 150 \times 10^{-34}\ \mathrm{J\ s\ cycle^{-1}}$$

$$\hbar = 1.054\ 571\ 818 \times 10^{-34}\ \mathrm{J\ s\ radian^{-1}}$$

The Planck constant is action per angular degree of freedom.

$$h/(\mathrm{J\ s\ radian^{-1}}) = [h/(\mathrm{J\ s\ cycle^{-1}})]/2\pi$$

$$\boxed{\text{Numerical value of PC in J s radian}^{-1}} = \boxed{\text{Numerical value of PC in J s cycle}^{-1}}/2\pi$$

In the SI one writes:

cycle^{-1} and radian^{-1} omitted in SI

h = 6.626 070 150 x 10^{-34} J s cycle^{-1}

$\hbar$ = 1.054 571 818 x 10^{-34} J s radian^{-1}

Despite Dirac's comment about $\hbar$ we have $h = \hbar$

Numerical value of $\hbar$ in J s radian^{-1}	=	Numerical value of h in J s cycle^{-1}	/2π

In SI **we say** *$\hbar = h/2\pi$. We really have $\hbar = h$.*

In SI we use $\hbar$ to designate

The Planck constant /(J s radian^{-1})

h and $\hbar$ are each the value of the Planck constant. The former uses the units J s cycle^{-1}, and the latter uses the units J s radian^{-1}. Their numerical values are in the ratio 2π.

An idea!

hertz = cycle/sec,

so we could introduce

ħertz = radian/sec.

hertz = Hz

ħertz = Ħz

We are now going to look at how we measure *h*

Phys. Rev., 7, 355 (1916)

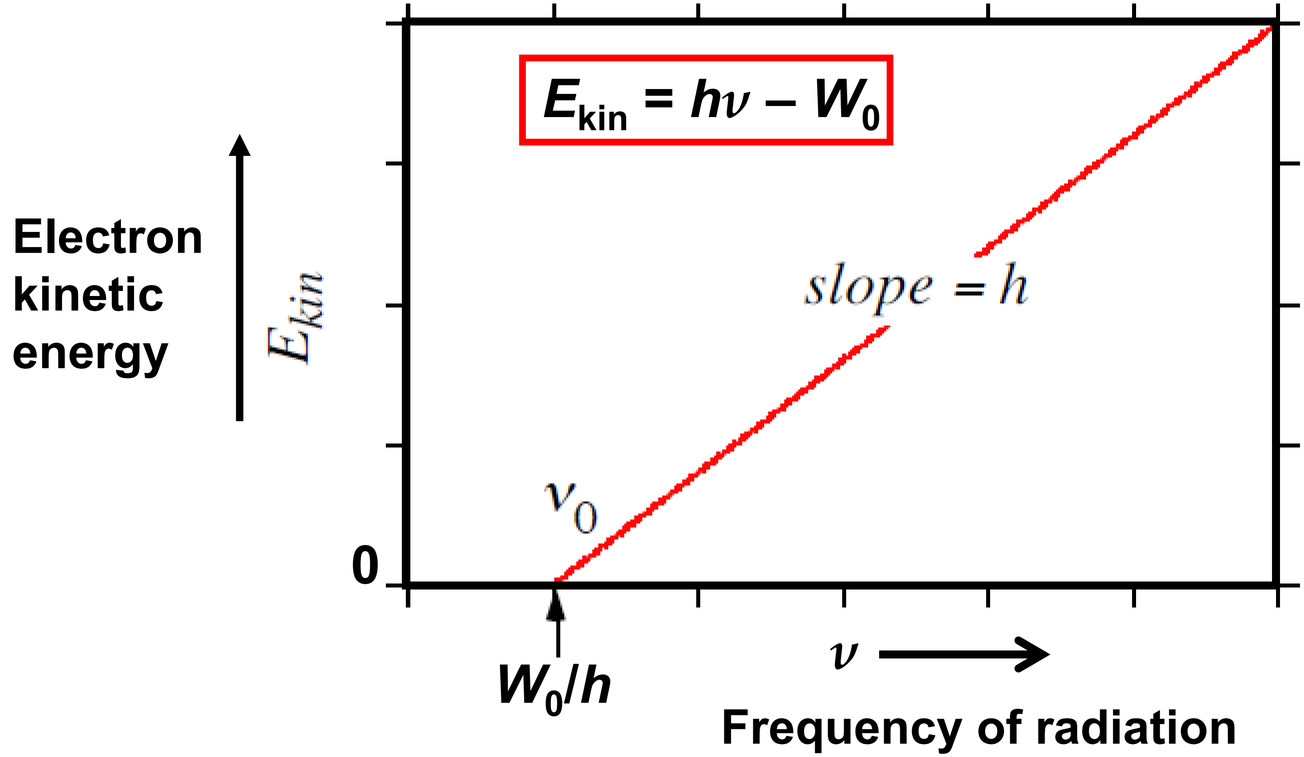
Robert Millikan
1868-1953

Millikan (Phys. Rev., 7, 355 (1916)) was the first person to make a good determination of the value of the Planck constant. He used the photoelectric effect. He wanted to test Einstein's theory that the energy of a photon is given by

$$E = h\nu$$

Millikan thought Einstein's theory was wrong, and he wanted to prove Einstein wrong.

Einstein's theory, $E(\text{photon}) = h\nu$, leads to the prediction of the photoelectron kinetic energy as a function of ν:



W_0 is the work function: The energy needed to remove an electron.

$W_0 = h\nu_0$

For all materials the slope of E_{kin} vs frequency has slope h.

Millikan separates Einstein's theory, $E(\text{photon}) = h\nu$, from the "Einstein equation" $E_{kin} = h\nu - W_0$. He verifies the Einstein equation, but questions Einstein's theory.

Millikan's paper describes 10 years of work in which he used cleaned lithium and sodium surfaces (in vacuo) with seven different mercury lines (from 2399 Å to 5461 Å) to measure electron energy as a function of frequency. He confirmed Einstein's equation .

Towards the end of his paper Millikan writes:

9. Theories of Photo Emission.

Perhaps it is still too early to assert with absolute confidence the general and exact validity of the Einstein equation. [redacted]

$$E_{kin} = h\nu - W_0$$

[redacted] the semi-corpuscular theory by which Einstein arrived at his equation seems at present to be wholly untenable.

Millikan's grudging summary:

10. SUMMARY.

1. Einstein's photoelectric equation has been subjected to very searching tests and $\boxed{\text{it appears}}$ in every case to predict exactly the observed results.

2. Planck's h has been photoelectrically determined with a precision of about .5 per cent. and is found to have the value

$$h = 6.57 \times 10^{-27}.$$

NO UNITS GIVEN
(it is in cgs units)

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO.

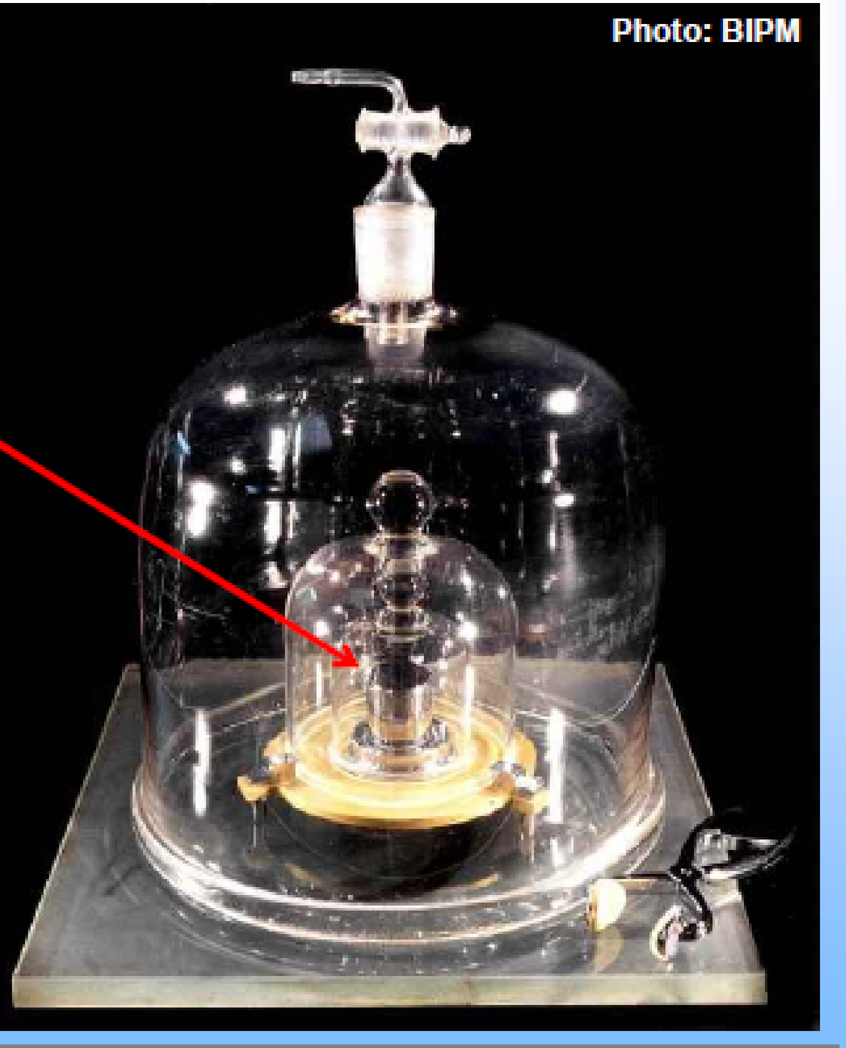
Now to the last measurement of the Planck constant. Used in the 2019 redefinition of the kilogram

1889-2019 definition of the kilogram in the SI

**The kilogram is the unit of mass;
it is equal to the mass of the international prototype of the kilogram.**

- represents the mass of 1 dm^3 of H_2O at maximum density (4 °C)
- manufactured around 1880, ratified in 1889
- alloy of 90% Pt and 10% Ir
- cylindrical shape, $Ø = h \sim 39$ mm
- kept at the BIPM in ambient air

The kilogram is the last SI base unit defined by a material artefact.



Two highly precise experimental methods for determining the Planck constant

Using a Kibble balance (improved Watt balance) with a standard 1 kg mass

X-ray crystal density

X-ray crystal density

One determines the number of Silicon atoms in a highly polished sphere of ^{28}Si with a mass of one kilogram.

h is proportional to $1/N_A$

Achim Leistner at the Australian Centre for Precision Optics (ACPO) holding a one-kilogram single-crystal silicon sphere prepared for the International Avogadro Coordination

Results from several labs since 2011 tabulated in Metrologia 55(2018) L13-L16

Use quantity calculus to obtain an easy to understand comparison of the results

$$h = 6.626\ 070\ 15 \times 10^{-34}\ \mathrm{J\ s}$$

$$h/(10^{-34}\ \mathrm{J\ s}) = 6.626\ 070\ 15$$

$$h/(10^{-34}\ \mathrm{J\ s}) - 6.626\ 0 = 0.000\ 070\ 15$$

$$[h/(10^{-34}\ \mathrm{J\ s}) - 6.626\ 0] \times 10^{5} = 7.015$$

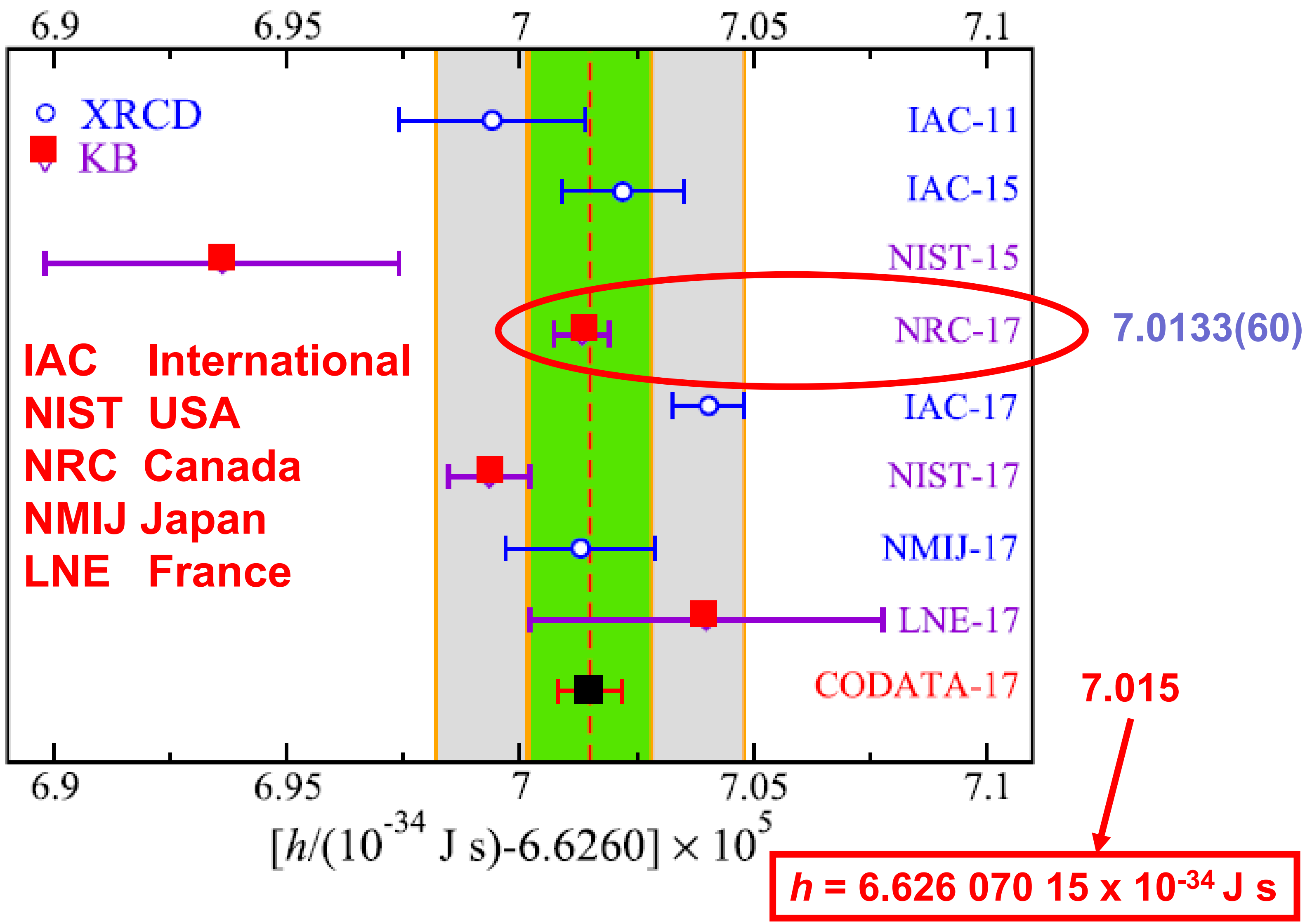


Figure 1. Values of the Planck constant h inferred from the input data in table 1 and the CODATA 2017 value in chronological order from top to bottom. The inner green band is ± 20 parts in 10^9

Metrologia 54 (2017) 399–409

NRC-17

https://doi.org/10.1088/1681-7575/aa70bf

A summary of the Planck constant determinations using the NRC Kibble balance

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Published 30 May 2017

The last determination of the numerical value of the Planck constant! Millikan’s paper was the first.

After fixing the numerical value of *h*, the Kibble balance will become an instrument for measuring absolute mass

The kilogram artifact in France will be dispensed with, although we could then determine its constancy.

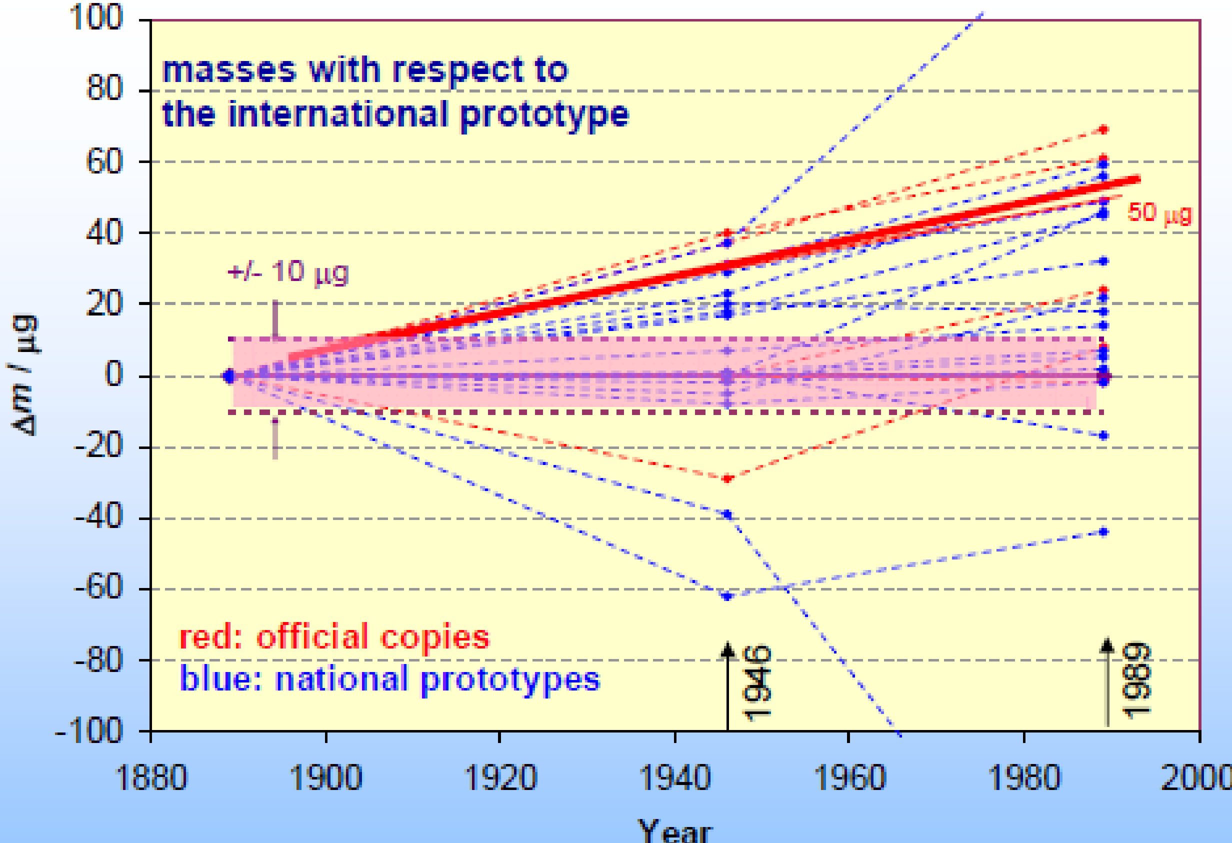
The Planck constant is not the only fundamental constant to get a defined numerical value in 2019.

Quantity	Value	Redefined base unit
h	$6.626\,070\,15 \times 10^{-34}$ J s	kg
e	$1.602\,176\,634 \times 10^{-19}$ C	ampere
k	$1.380\,649 \times 10^{-23}$ J K^{-1}	K
N_A	$6.022\,140\,76 \times 10^{23}$ mol^{-1}	mole

See www.bipm.org

EXTRA SLIDES FOR QUESTIONS

Calibration history of the oldest national prototypes



Variations of about **50 µg (5×10^{-8}) in the mass of the standards over 100 years, that is 0.5 µg / year**

Masses of same material can be compared to within 1 µg

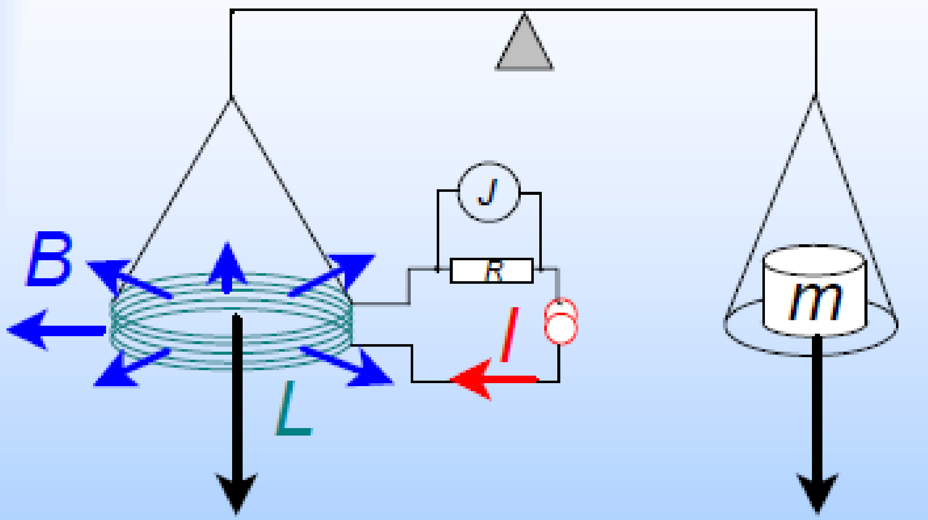
A drifing kg also influences the electrical units

Is the IPK losing mass or are the check standards getting heavier ? ?

► **Redefinition of the kg in terms of a fundamental constant of nature, for example Planck constant *h* (advantageous for electrical metrology)**

Kibble balance principle - 1

Phase 1: static experiment



$F_{el} = I\,L\,B$

$F_m = m\,g$

Weight of a test mass is compared with the force on a coil in a magnetic field.

In a radial magnetic field,

$$m\,g = I\,L\,B$$

current

wire length

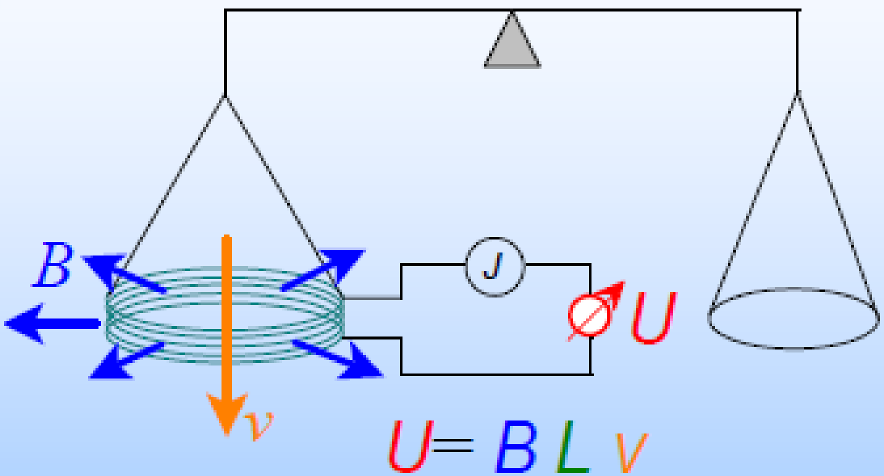
flux density

Measure *I* needed for balance

Kibble balance principle - 2

Phase 2: dynamic experiment

Coil is moved through the magnetic field and a voltage is induced.



In a radial magnetic field,

$$\boxed{U = B\,L\,v}$$

ind. voltage

flux density

wire length

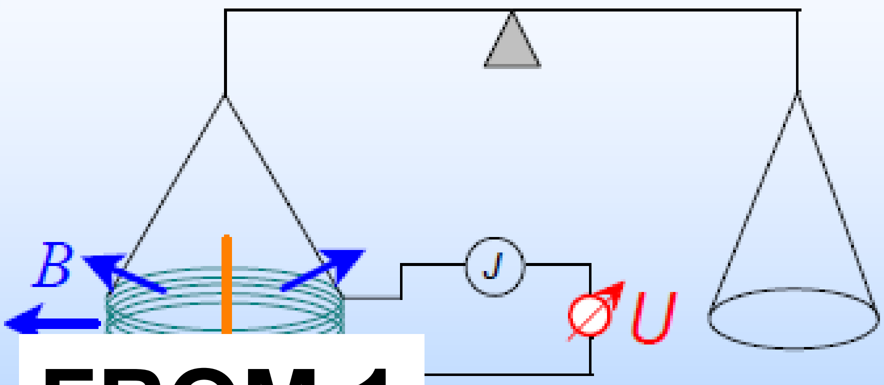
velocity

Measure *U* and *v*

Kibble balance principle - 2

Phase 2: dynamic experiment

Coil is moved through the magnetic field and a voltage is induced.



In a radial magnetic field,

FROM 1

$$mg = ILB$$

current — wire length — flux density

$$U = BLv$$

ind. voltage — flux density — wire length — velocity

$$\boxed{BL = mg/I = U/v}$$

From exp 2 From exp 1

$\boxed{BL = mg/I = U_1/v} \rightarrow \boxed{mgv = U_1 I}$

The mass m is a 1 kg or 0.5 kg standard mass

Measure force difference between gravitational and em forces using a comparator

Measure local acceleration due to gravity, *g* using a gravimeter

Measure the velocity of the coil, *v* using interferometer and Cs clock

Measure voltage U_1 using the Josephson effect

$U_1 = h\,\nu_1/(2\,e)$. ***U* to ν conversion.**

Measure current *I* by measuring *R* using the quantum Hall effect

$I = U_2/R = e\,\nu_2\,/\,2.$

$$\boxed{mgv = h\,\nu_1\ \nu_2\,/\,4}$$

$$\boxed{h = 4\,mgv\,/(\nu_1\ \nu_2)}$$

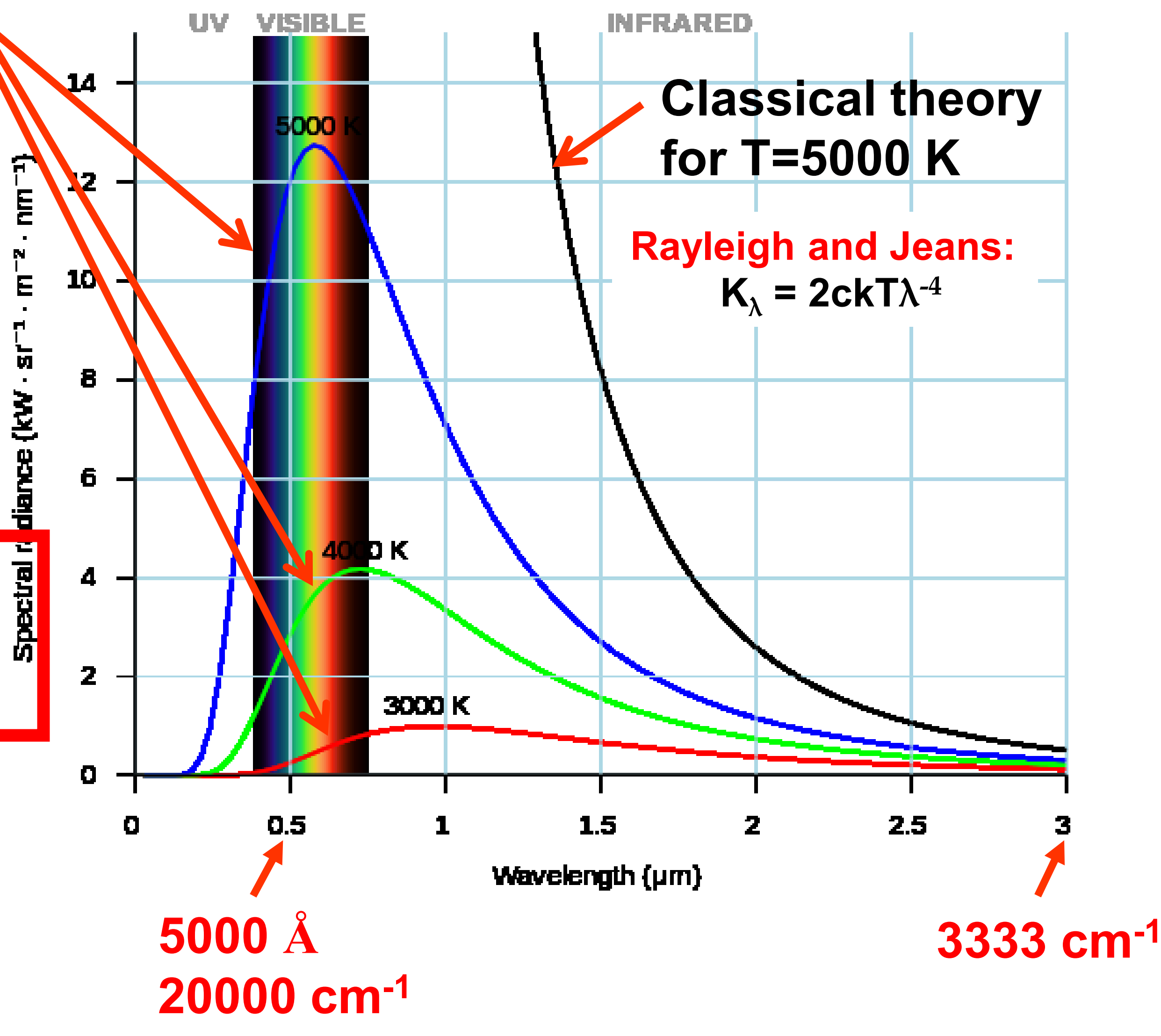
Spectral Distribution Function $I(\lambda,T) = K_\lambda$ of Black-Body Radiation. The Wien expression.

experiment

First experiment
Langley 1886

More experiments
Paschen and Wien
1896-1899

Wien gets empirical expression:
$K_\lambda = b\lambda^{-5}/[\exp(a/\lambda T)]$



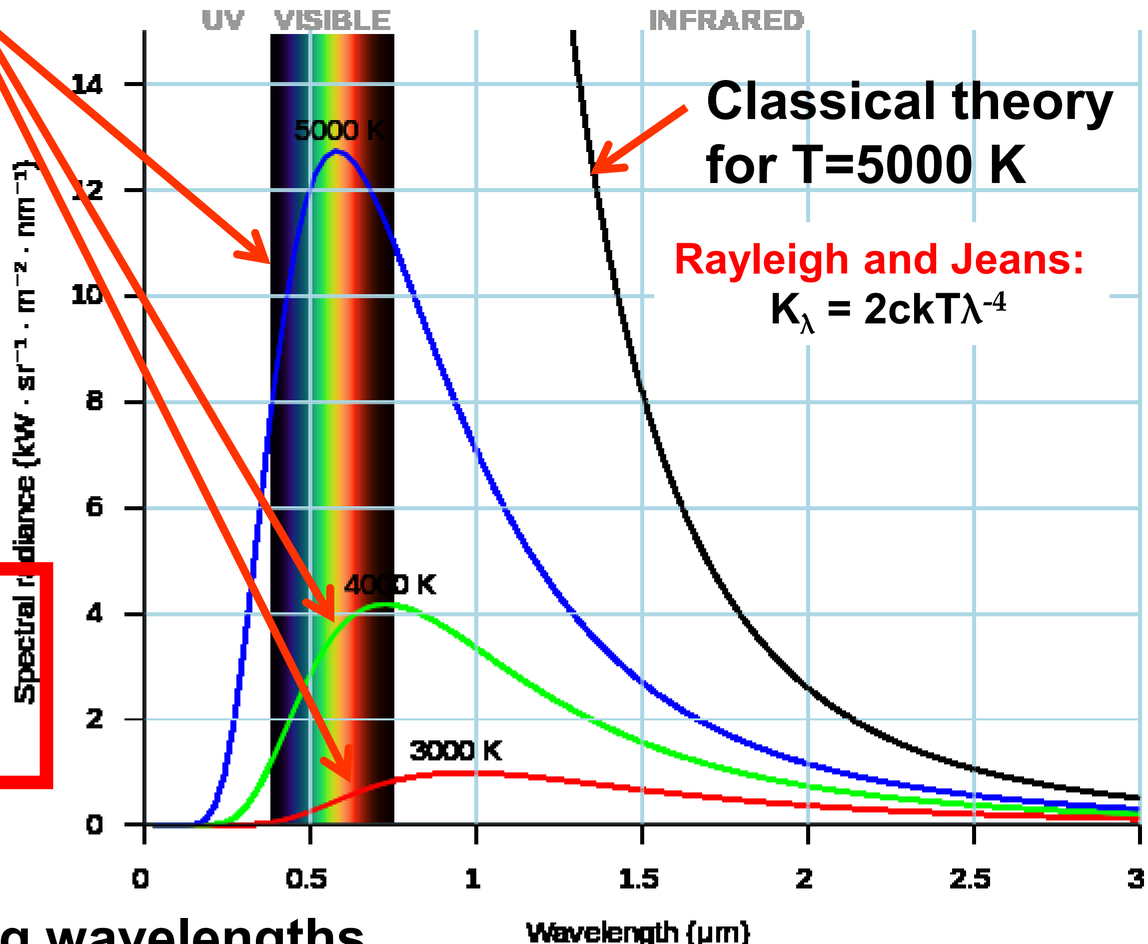
Spectral Distribution Function $I(\lambda,T) = K_\lambda$ of Black-Body Radiation. Wien is no good at large λ.

experiment

First experiment
Langley 1886

More experiments
Paschen and Wien
1896-1899

Wien gets empirical expression:
$K_\lambda = b\lambda^{-5}/[\exp(a/\lambda T)]$



Classical theory
for T=5000 K

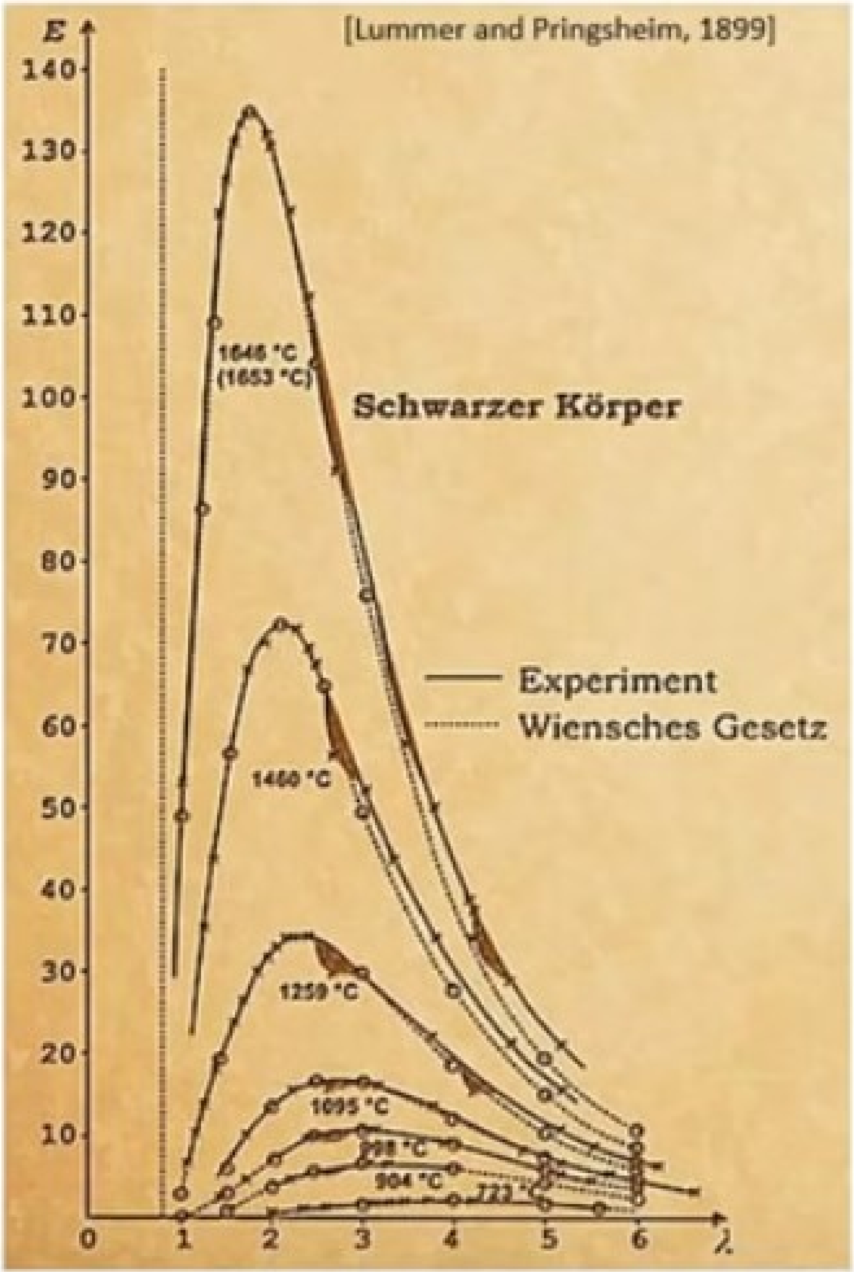
Rayleigh and Jeans:
$K_\lambda = 2ckT\lambda^{-4}$

Wien no good at long wavelengths

At large λ fits $K_\lambda = 2ckT\lambda^{-4}$

1899 Lummer, Pringsheim, Rubens, and Kurlbaum experiments to 50 μm

Max Planck (1900): Fits Black Body Radiation



IV.

Verhandlungen der physikal. Gesellschaft 2, p. 202; 1900.

Ueber eine Verbesserung der Wienschen Spektralgleichung. (1)

(Vorgetragen in der Sitzung vom 19. Oktober 1900.)

$$e_\lambda = \frac{C\lambda^{-5}}{e^{c'/\lambda T} - 1}$$

Planck 'adds' this "-1" to the denominator of Wien's expression. Fits at high λ.

AN EMPIRICAL FIT

Max Planck(1900): DERIVES his Distribution Law by assuming Black Body "oscillators" are quantized

"Oscillators" in the BB are quantized

$E = h\nu$

The birth of the Planck constant

Leads to

Planck's Distribution Law

$e_\lambda = C\lambda^{-5} / [\exp(c'/\lambda T) - 1]$

$C = 2hc^2$

$c'/\lambda T = h\nu/kT$

At large λ: $\exp(c'/\lambda T) = 1 + (c'/\lambda T)$, and we get

Rayleigh-Jeans classical expression

$e_\lambda = 2ckT\lambda^{-4}$

Einstein (1905): EM radiation is Quantized

1879-1955

Einstein's famous paper with the title:

"On a heuristic viewpoint concerning the production and transformation of light."

Introduces photons with energy $E = h\nu$

"It endowed Planck's quantum hypothesis with physical reality. The oscillators for which Planck proposed energy quantization were fictitious, and his theory for blackbody radiation lacked obvious physical consequences. But the radiation theory for which Einstein proposed energy quantization was real, and his theory had immediate physical consequences."

D. Kleppner, Physics Today, February 2005 page 30.

Landau and Lifshitz, Page 20
Quantum Mechanics

Wave optics and geometrical optics

On the basis of this analogy, we can assert that the phase ϕ of the wave function, in the limiting (classical) case, must be proportional to the mechanical action S of the physical system considered, i.e. we must have $S = \text{constant} \times \phi$. The constant of proportionality is called *Planck's constant*† and is denoted by $\hbar$. It has the dimensions of action (since ϕ is dimensionless) and has the value

But not unit-less

$$\hbar = 1{\cdot}054 \times 10^{-27} \text{ erg sec. radian}^{-1}$$

Thus, the wave function of an "almost classical" (or, as we say, *quasi-classical*) physical system has the form

$$\Psi = a e^{iS/\hbar}. \tag{6.1}$$

$e^{ix} = \cos x + i \sin x$ x is the numerical value of the angle in radians

Thus $S/\hbar$ has to be numerical value in radians. S is numerical value in erg s, so we must use numerical value of the Planck constant in erg s radian^{-1}.

Landau and Lifshitz do not explain why they use the numerical value of $\hbar$.

† It was introduced into physics by M. Planck in 1900. The constant $\hbar$, which we use everywhere in this book, is, strictly speaking, Planck's constant divided by 2π; this is Dirac's notation.

Everything in red added by us in this image.

In the Path Integral Formulation of QM we need the quantum of action to be in J s radians^{-1}

The Path Integral Formulation states that the transition amplitude is the integral of exp(i*S*/*ħ*) over all possible routes from the initial to the final state. *S* is the classical action (in J s).

Because it is the argument of exp, *S*/*ħ* has to be the numerical value of the quantity in radians. *S* is in J s, so *ħ* has to be *h* in J s radian^{-1}