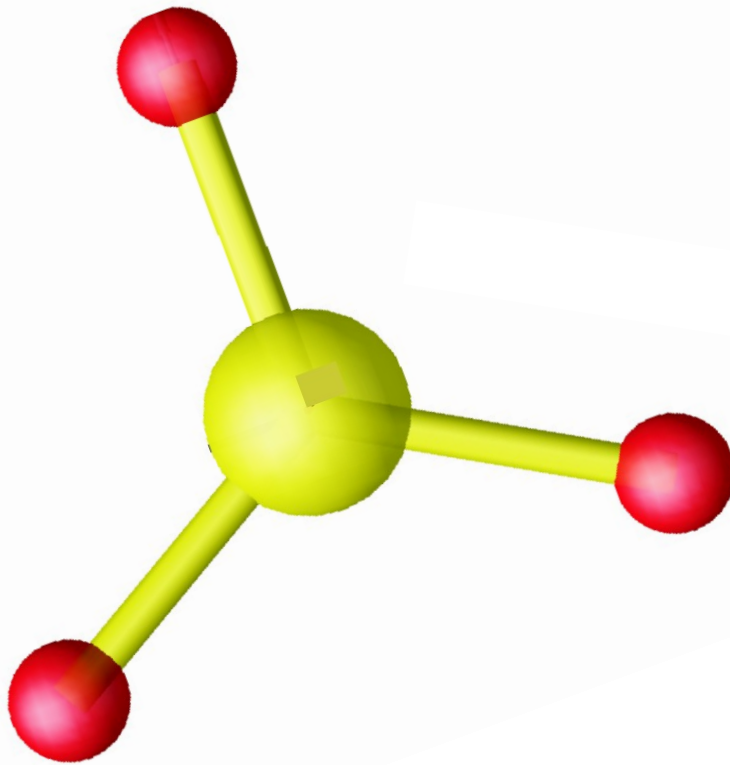




- Forbidden rotational spectrum
- Rovibrational energy cluster formation



SO₃ calculations

- Nuclear-motion calculations carried out with TROVE¹
- Potential-energy surface obtained by *ab initio* methods²

¹S.N. Yurchenko, W. Thiel, and P. Jensen, *J. Mol. Spectrosc.* **245**, 126 (2007).

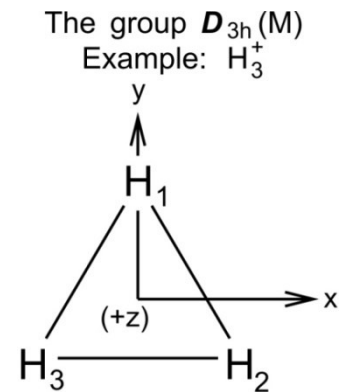
²D. S. Underwood, J. Tennyson, and S. N. Yurchenko, *Phys. Chem. Chem. Phys.* **15**, 10118 (2013).

SO₃: All nuclei are zero-spin bosons

Only levels with
 $\Gamma_{\text{rve}} = A'_1$ or A'_1
 exist in Nature

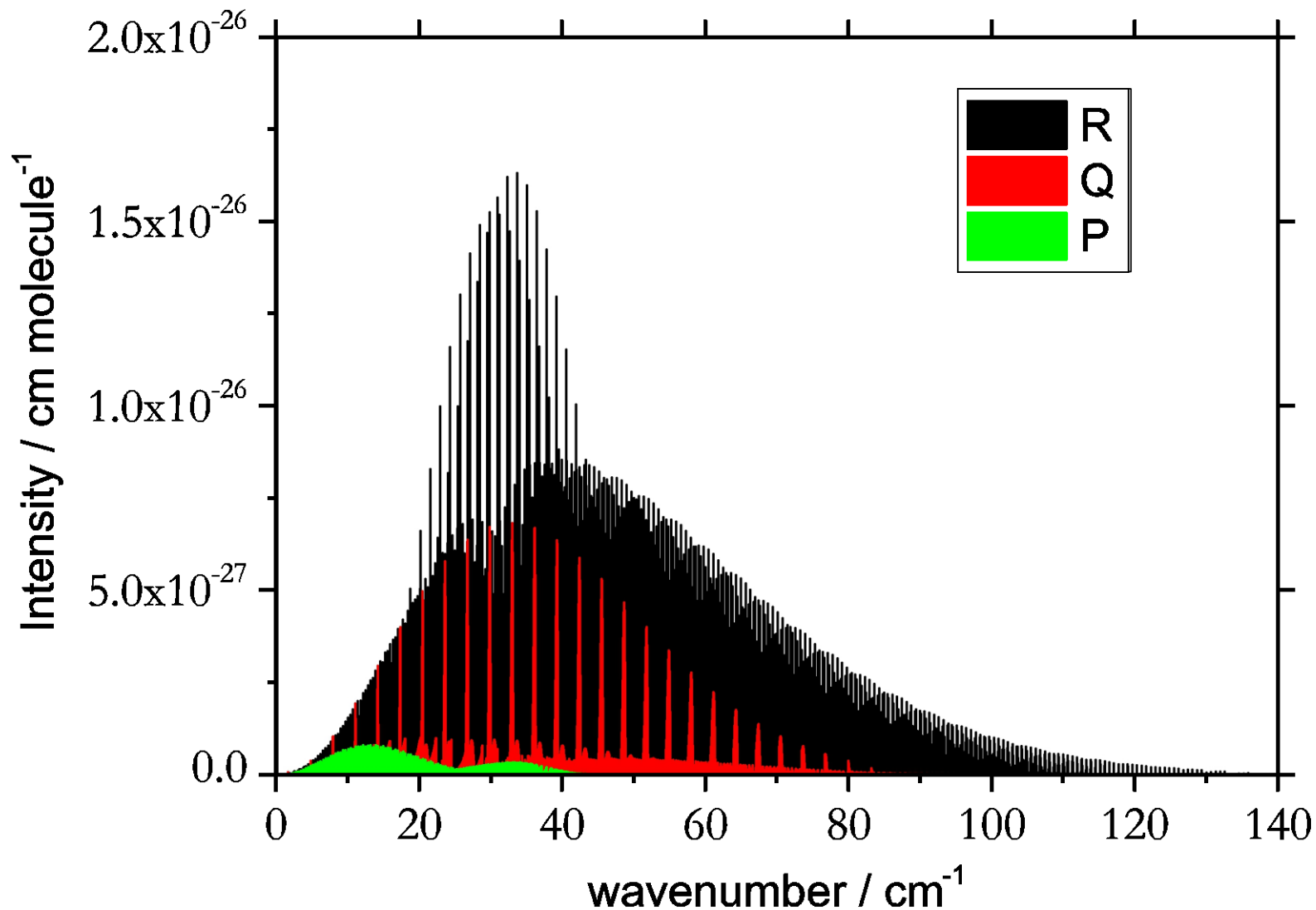
All other
 levels are
 missing!

$K = 3t$ in vib.
 ground state



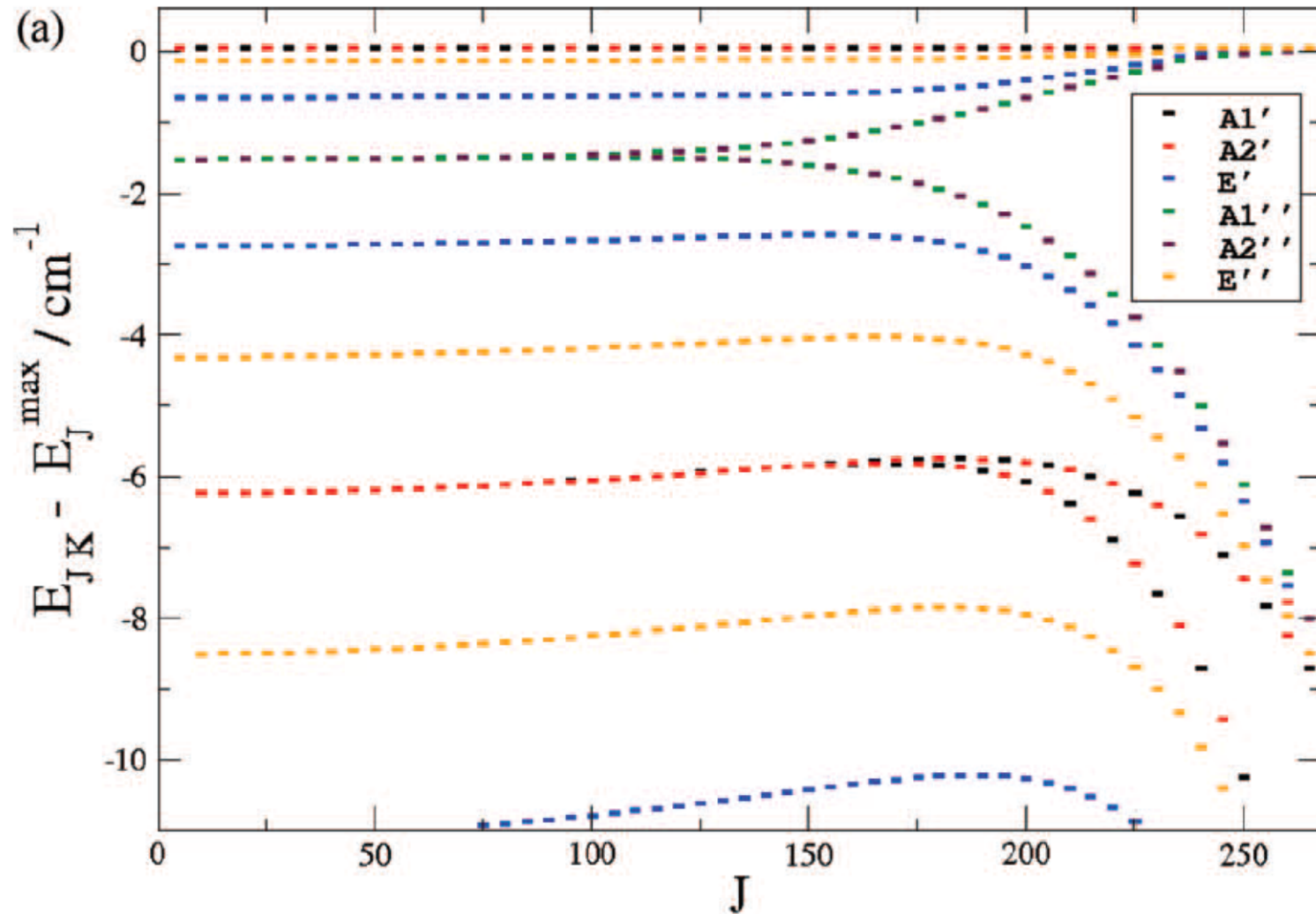
	E	(123)	(23)	E^*	$(123)^*$	$(23)^*$	
$D_{3h}(M)$:	1	2	3	1	2	3	
D_{3h} :	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$	
Equiv. rot.:	R^0	$R_z^{2\pi/3}$	$R_{\pi/2}^\pi$	R_z^π	$R_z^{5\pi/3}$	R_0^π	
A'_1 :	1	1	1	1	1	1	$\alpha_{zz}, \alpha_{xx} + \alpha_{yy}$
A''_1 :	1	1	1	-1	-1	-1	$\Gamma(\mu_A)$
A'_2:	1	1	1	1	1	1	$\hat{J}_z$
A''_2:	1	1	-1	-1	-1	1	T_z
E':	2	-1	0	2	-1	0	$(T_x, T_y),$
E'':	2	-1	0	-2	1	0	$(\alpha_{xx} - \alpha_{yy}, \alpha_{xy})$
							$(\hat{J}_x, \hat{J}_y),$
							$(\alpha_{xz}, \alpha_{yz})$

SO₃ Forbidden rotational spectrum



SO₃ Reduced energies

Vibrational ground state



Sixfold energy clusters!

Confirmation: Rotational energy surface¹

Energy as function of angles
 θ and ϕ that define the
classical angular momentum

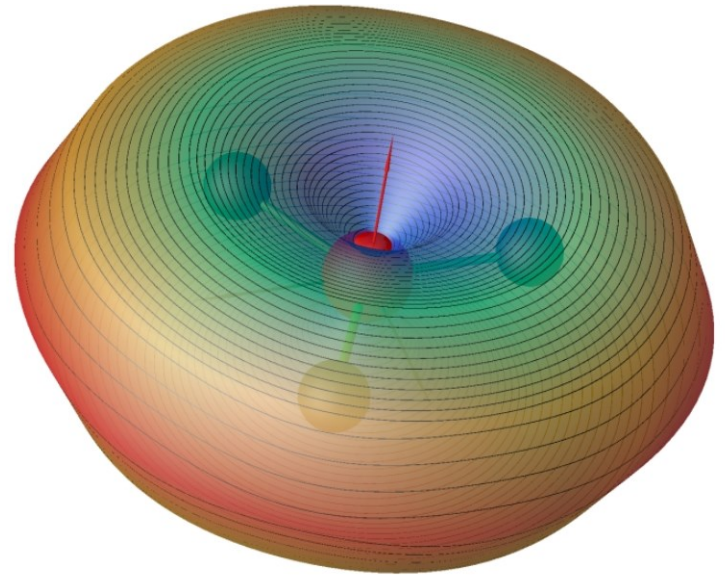
$J = 200$

$$J_x = \sqrt{J(J+1)} \hbar \sin \theta \cos \phi,$$

$$J_y = \sqrt{J(J+1)} \hbar \sin \theta \sin \phi,$$

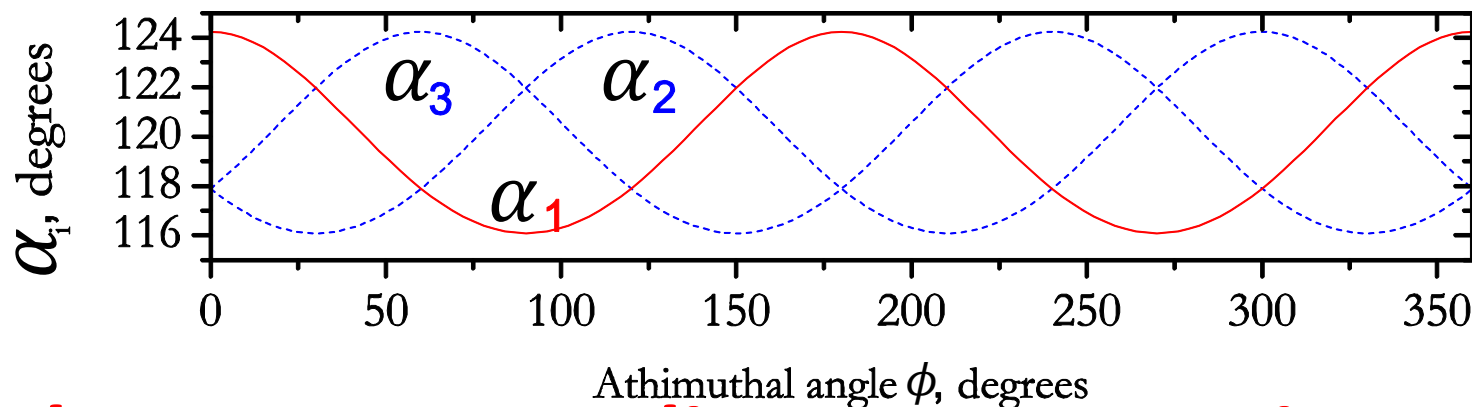
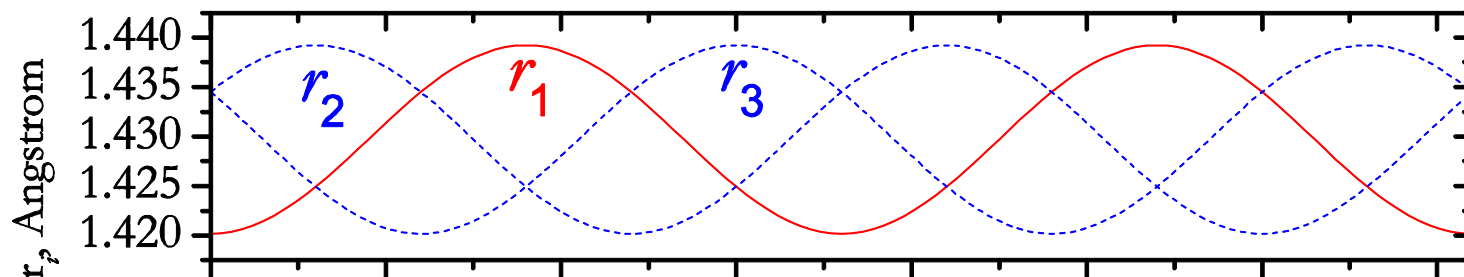
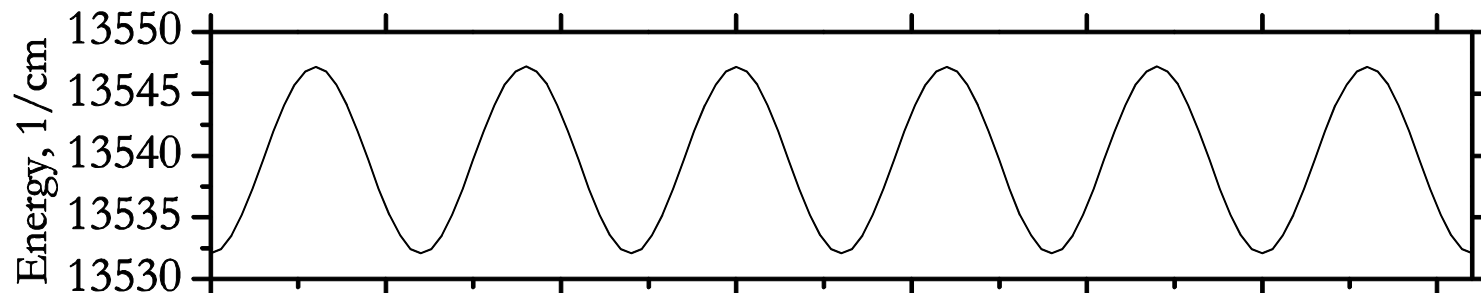
$$J_z = \sqrt{J(J+1)} \hbar \cos \theta,$$

in the molecule-fixed axis system.



¹W. G. Harter, Phys. Rev. A **24**, 192 (1981).

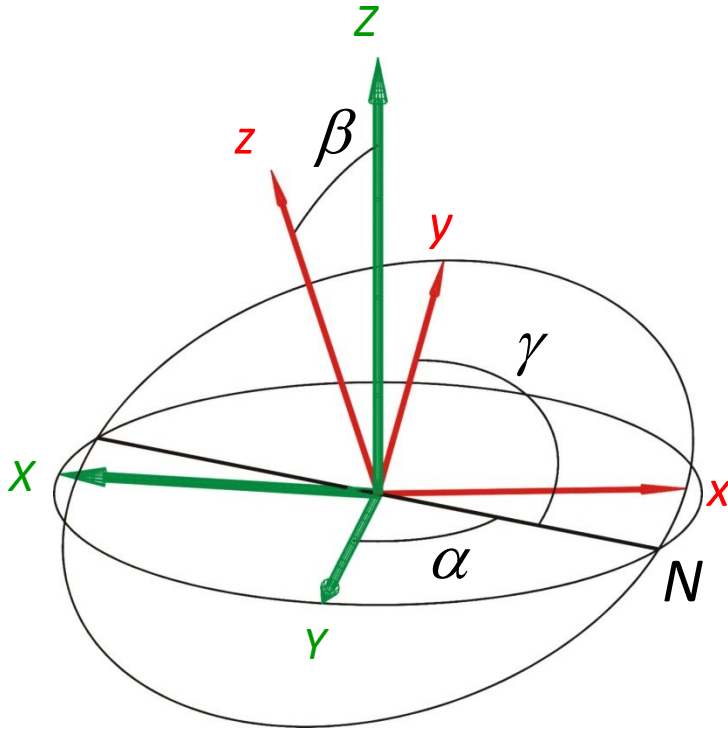
Six maxima along the equator, $\theta = 90^\circ$



**Angular momentum lies on equator in
cluster states!**

$$F(\beta, \gamma) = \int_0^{2\pi} d\alpha \int_{\text{vib}} dV (\psi_n^{\text{PCS}})^* \psi_n^{\text{PCS}},$$

xyz is molecule-fixed; **XYZ** is space-fixed



$F(\beta, \gamma)$ is the probability density of β, γ or of **Z axis** relative to molecule

- (α, β, γ) define orientation of molecule (**xyz**) relative to laboratory (**XYZ**).
- (β, γ) define orientation of **Z axis** relative to molecule (**xyz**).

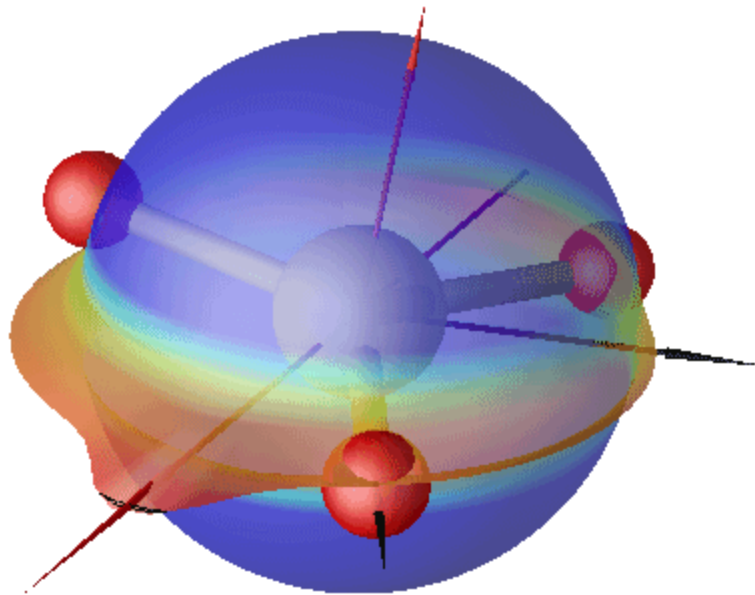
$$F(\beta, \gamma) = \int_0^{2\pi} d\alpha \int_{\text{vib}} dV (\psi_n^{\text{PCS}})^* \psi_n^{\text{PCS}},$$

ψ_n^{PCS} is “Primitive Cluster State”

- A cluster is described by six symmetrically equivalent PCS states
- The correspondence between PCS states and symmetrized states can be determined, for example, by projection operator methods

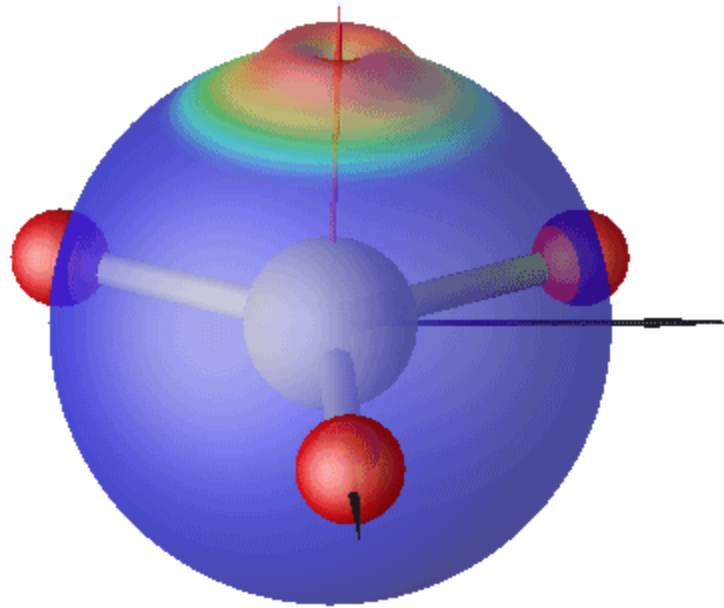
We consider states with
 $m = J$ to align Z axis
 with J

$$F(\beta, \gamma) \text{ for } ^{32}\text{S}^{16}\text{O}_3, J = m = 250$$



Angular momentum (rotation axis) lies on equator!

Contrast: $F(\beta, \gamma)$ for $^{32}\text{S}^{16}\text{O}_3$, $(J, K) = (100, 99)$ – at bottom of J manifold.



***Angular momentum (rotation axis) lies
towards north pole!***



- Rovibrational energy clusters determined by QM calculations
- Surprising, but borne out by semiclassical RES theory

