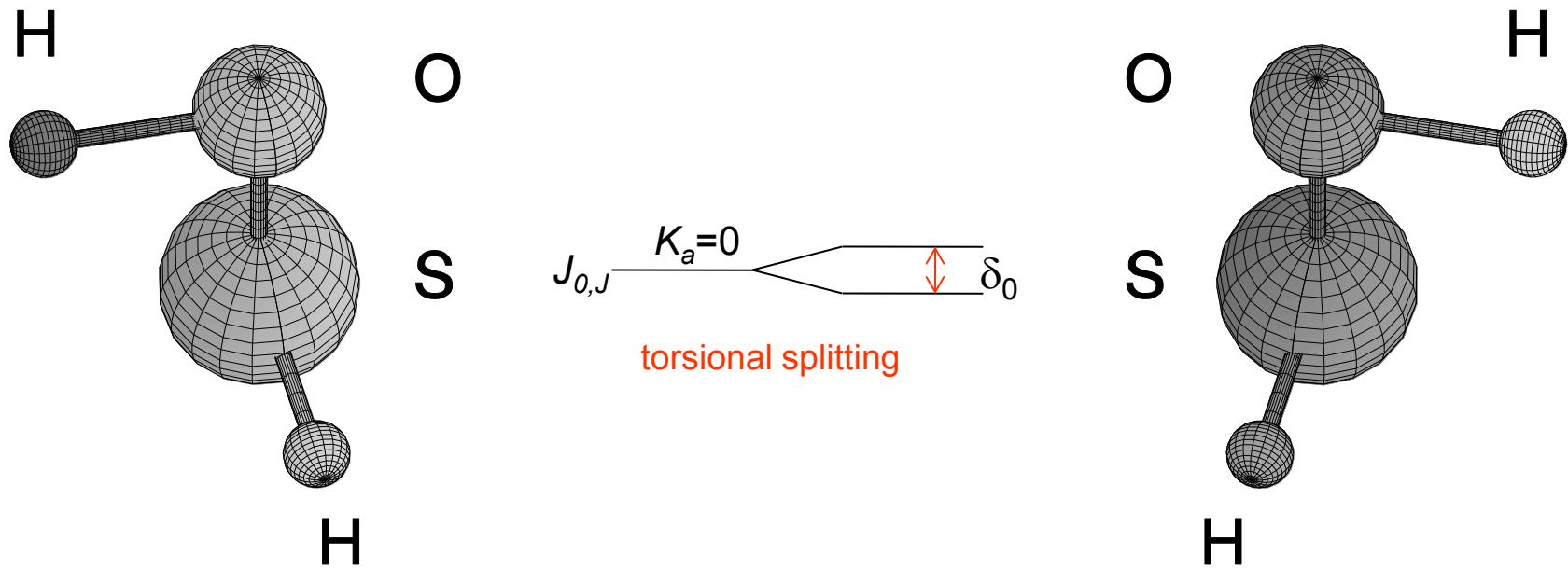
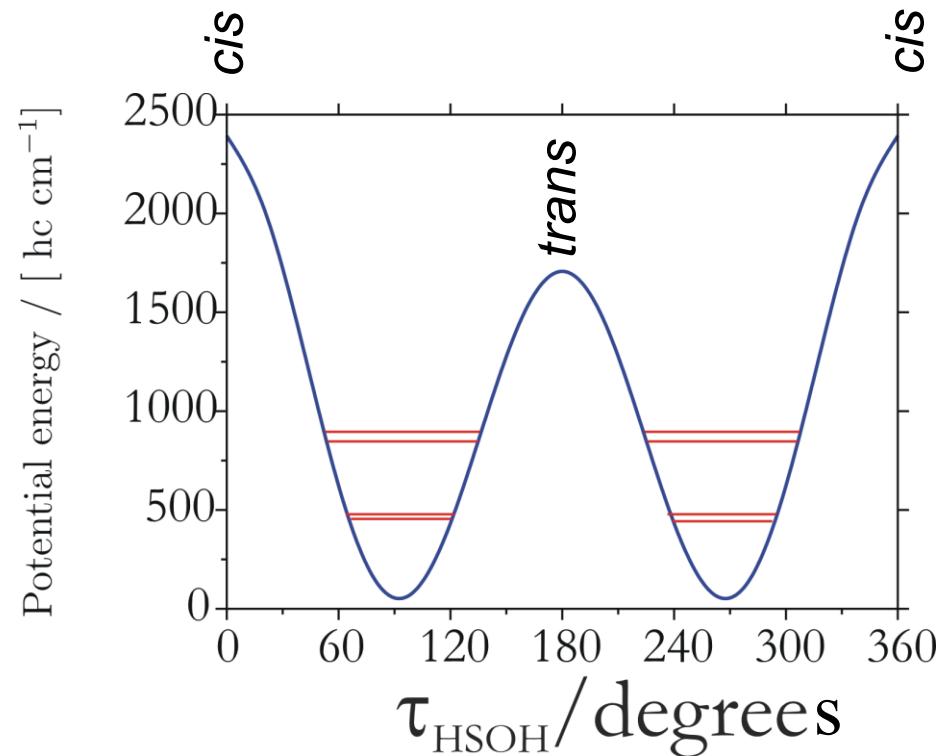
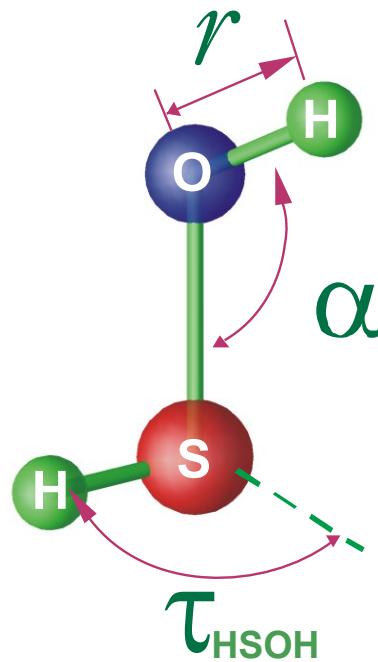


## *Torsional splittings and anomalous intensities in HSOH*

- Two enantiomer minima
- Internal rotation about the SO bond, strongly coupled to the rotation about the *a* axis (associated with  $K_a$ )
- Torsional splittings



## HSOH: Torsional potential



Torsional splittings show  
strong variation with  $K_a$

## HOOH, HSSH, HNCNH: Splittings “stagger” with $K_a$

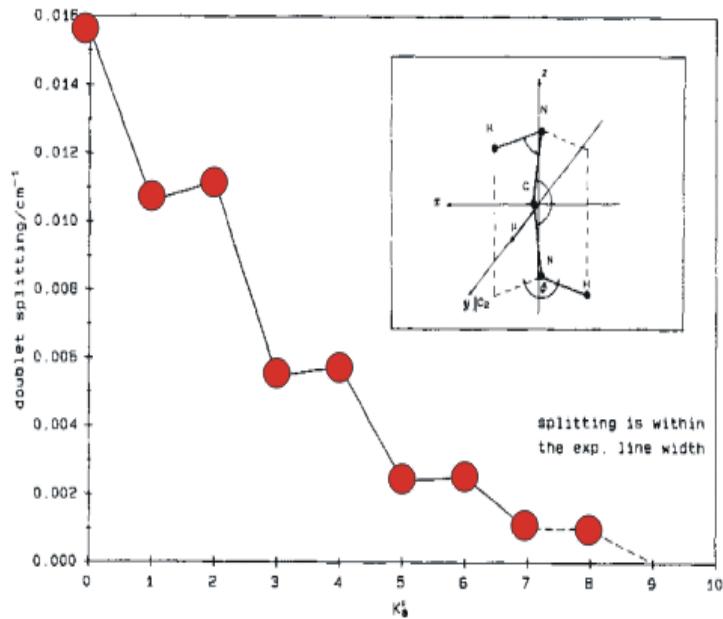


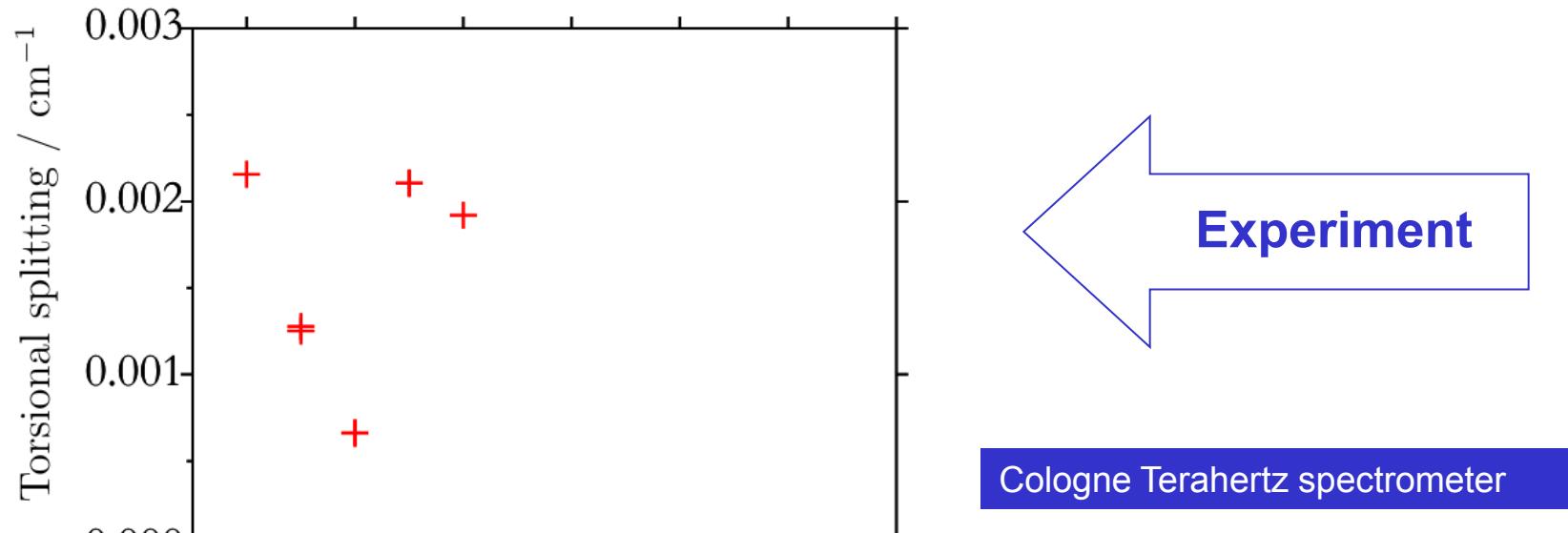
FIG. 13. Dependence of the torsional doublet splitting on  $K_a$ . The doublet splitting averaged over all  $J$ -values was obtained from the line positions of the ' $Q_0$ ' branch for  $K_a = 0$ , the ' $R_{K_a}$ ' branch for  $K_a = 1$  to  $K_a = 4$ , the ' $R_{K_a}$ ' and ' $Q_{K_a}$ ' branches for  $K_a = 5$  and  $6$ , and from an analysis of the linewidth for  $K_a = 7$ . For  $K_a = 8$  the splitting is identical to that for  $K_a = 7$  within the experimental accuracy. For  $K_a = 9$ , no broadening of the ' $R_{K_a}$ ' lines due to torsional doubling could be detected.

Example: HNCNH

Experiment: M. Birk, M. Winnewisser,  
*J. Mol. Spectrosc.* **136**, 402 (1989)

Semi-empirical explanation by Hougen  
and co-workers for HOOH and HSSH:  
J.T. Hougen, *Can. J. Phys.* **62**, 1392 (1984).  
J.T. Hougen, B.M. DeKoven, *J. Mol. Spectrosc.* **98**, 375 (1983).

## HSOH: No staggering, more complicated variation



First observation (for  $K_a \leq 3$ ) in 2003

M. Behnke, J. Suhr, S. Thorwirth, F. Lewen, H. Lichau, J. Hahn, J. Gauss, K.M.T. Yamada, G. Winnewisser, *J. Mol. Spectrosc.* **221**, 121 (2003)

G. Winnewisser, F. Lewen, S. Thorwirth, M. Behnke, J. Hahn, J. Gauss, and E. Herbst, *Chem. Eur. J.* **9**, 5501 (2003)

O. Baum, M. Koerber, O. Ricken, G. Winnewisser, S. N. Yurchenko, S. Schlemmer, K. M. T. Yamada, and T. F. Giesen, *J. Chem. Phys.* **129**, 224312 (2008).

## Approach 0 (2004): Semi-empirical, following the ideas of Hougen:

J.T. Hougen, *Can. J. Phys.* **62**, 1392 (1984).

J.T. Hougen, B.M. DeKoven, *J. Mol. Spectrosc.* **98**, 375 (1983).

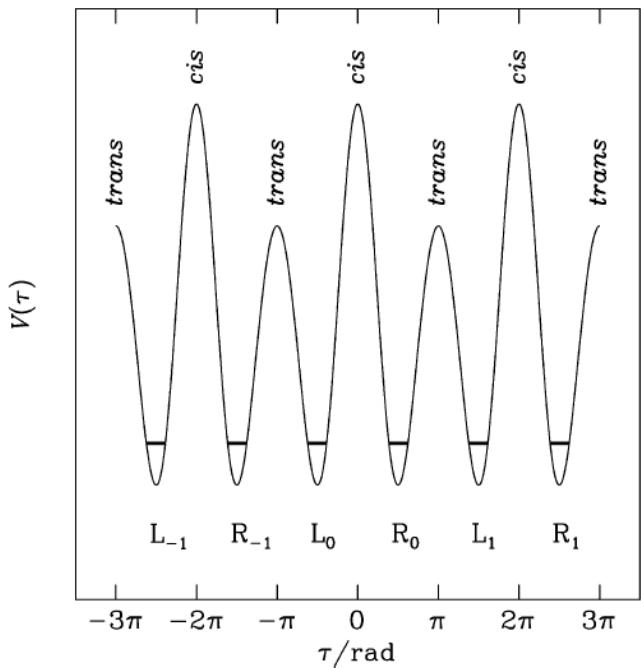
Ratio of  $I_a$  moments of inertia  $I_{\text{SH}}/I_{\text{total}} \approx 1/3$ .

Pretend that  $\tau_{\text{HSOH}} \in [-3\pi, 3\pi]$

Molecule formally has  $C_{3v}(M)$  symmetry

Do quantum mechanics under this assumption.

Subject results to „reality check“: A  $2\pi$  rotation of the SH moiety relative to OH moiety must leave wavefunction unchanged.



$$\tau_{HSOH} \in [-3\pi, 3\pi]$$

6 × 6 matrix:

$$\begin{pmatrix} Q & W_c & 0 & 0 & 0 & W_t \\ W_c & Q & W_t & 0 & 0 & 0 \\ 0 & W_t & Q & W_c & 0 & 0 \\ 0 & 0 & W_c & Q & W_t & 0 \\ 0 & 0 & 0 & W_t & Q & W_c \\ W_t & 0 & 0 & 0 & W_c & Q \end{pmatrix}$$

# Conclusion for HSOH (2009):

The torsional splittings of HSOH in the ground state for each  $K$ .

$K$	$\Delta_{\text{tor}}$	Observed <sup>a</sup> /MHz	Calculated <sup>b</sup> /MHz
0	$2 W_c + W_t $	64.5	63.9
1	$2 W_c + W_t/2 - D_{ab} $	37.8	33.3
2	$2\sqrt{(W_c - W_t/2 + 2D_{ab})^2 + 3W_t^2/4}$	52.1	50.9
3	$2 W_c + W_t - 9D_{12}^2/2(W_c + W_t) $	62.9	63.5
4	$2\sqrt{(W_c - W_t/2 - 4D_{ab})^2 + 3W_t^2/4}$	57.2	57.3
5	$2\sqrt{(W_c - W_t/2 + 5D_{ab})^2 + 3W_t^2/4}$	49.0	49.2
6	$2 W_c + W_t - 36D_{12}^2/2(W_c + W_t) $		62.1
7	$2\sqrt{(W_c - W_t/2 - 7D_{ab})^2 + 3W_t^2/4}$		61.6
8	$2\sqrt{(W_c - W_t/2 + 8D_{ab})^2 + 3W_t^2/4}$		48.6
9	$2 W_c + W_t - 81D_{12}^2/2(W_c + W_t) $		59.9
10	$2\sqrt{(W_c - W_t/2 - 10D_{ab})^2 + 3W_t^2/4}$		66.6

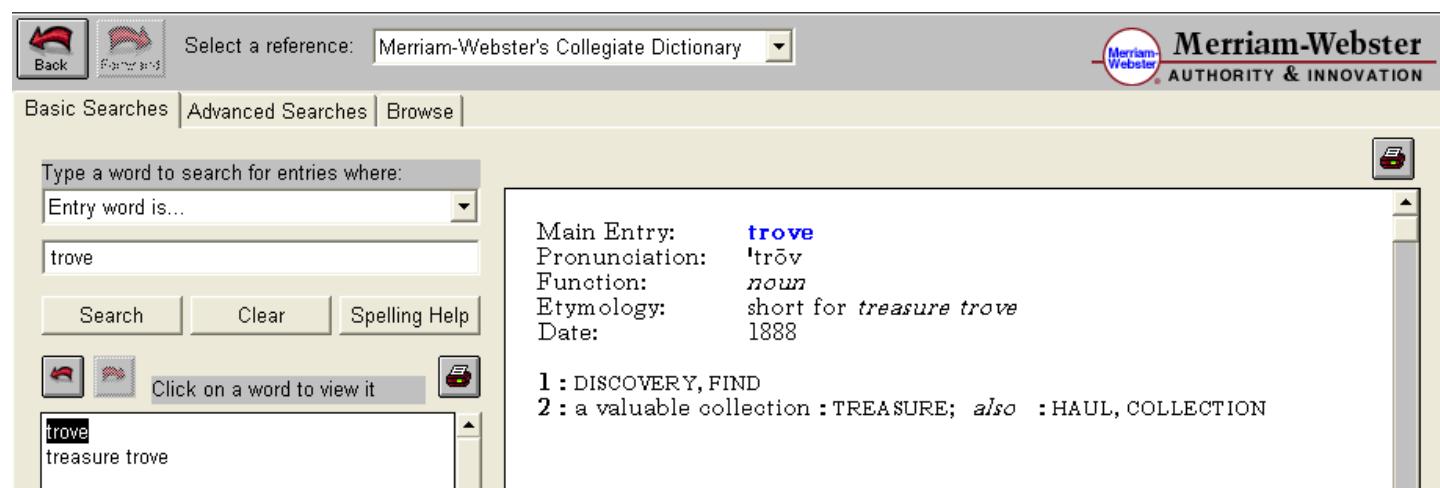
## Approach 1 (2008): *Ab initio* potential energy surface + TROVE calculation of torsional splittings

*Ab initio* potential energy surface:

- CCSD(T) method
- 105000 data points with aug-cc-pVTZ basis set,  
energies up to 20000 cm<sup>-1</sup> above equilibrium
- 10168 data points with aug-cc-PV(Q+d)Z basis set,  
energies up to 12000 cm<sup>-1</sup> above equilibrium
- Simultaneous, weighted fitting to all data points, 762  
parameters varied, standard error 2.8 cm<sup>-1</sup>

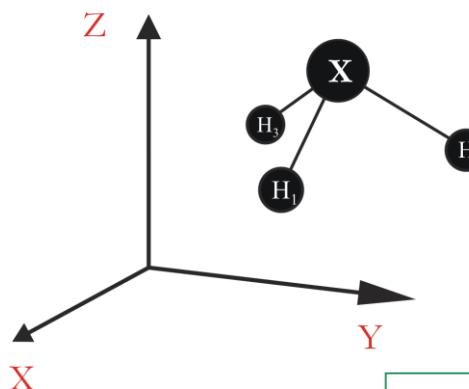
# TROVE?

**TROVE: Theoretical ROVibrational Energies:**  
Variational calculations of rotation-vibration for a general polyatomic molecule in an isolated electronic state



## Hamiltonian: Coordinate transformation

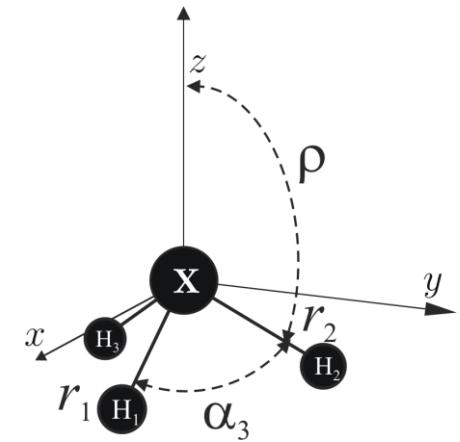
$$\left( \sum_{i=1}^N \frac{-\hbar^2}{2M_i} \nabla_i^2 + V \right) \Psi = E\Psi$$



Laboratory fixed  
Cartesian  
coordinate  
system

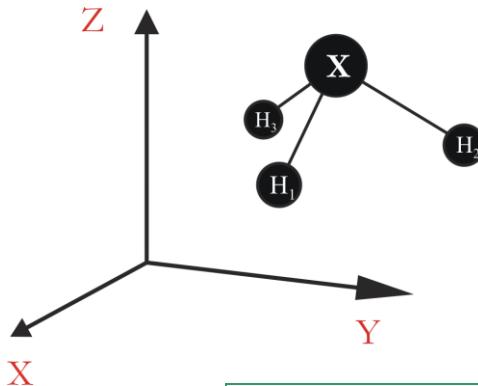
Body fixed internal  
coordinate system

$$\mathbf{R}_i = \mathbf{R}^{\text{CM}} + S^{-1}(\theta, \phi, \chi) [\mathbf{R}_i^{\text{MS}}(\mathbf{r})]$$



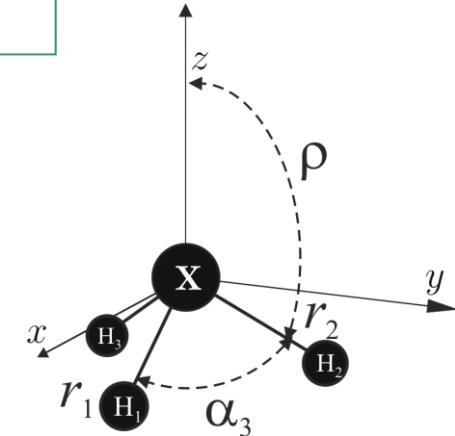
$$\left[ \frac{1}{2} \sum_{\lambda=1}^{3N} \sum_{\mu=1}^{3N} \Pi_{\lambda} G_{\lambda,\mu}(\mathbf{r}) \Pi_{\mu} + U(\mathbf{r}) + V(\mathbf{r}) \right] \Psi = E\Psi$$

$$\left( \sum_{i=1}^N \frac{-\hbar^2}{2M_i} \nabla_i^2 + V \right) \Psi = E\Psi$$



Laboratory fixed  
Cartesian  
coordinate  
system

Body fixed internal  
coordinate system



- The coordinate transformation is done by the program
- „One transformation for all molecules“

$$\left[ \frac{1}{2} \sum_{\lambda=1}^{3N} \sum_{\mu=1}^{3N} \Pi_\lambda G_{\lambda,\mu}(\mathbf{r}) \Pi_\mu + U(\mathbf{r}) + V(\mathbf{r}) \right] \Psi = E\Psi$$

## Coordinate transformation: Definitions

$$\left[ \frac{1}{2} \sum_{\lambda=1}^{3N} \sum_{\mu=1}^{3N} \Pi_{\lambda} G_{\lambda,\mu}(\mathbf{r}) \Pi_{\mu} + U(\mathbf{r}) + V(\mathbf{r}) \right] \Psi = E\Psi$$

**Kinetic energy factor**

$$G_{\lambda,\mu} = \sum_{\alpha=x,y,z} \sum_{i=1} \frac{s_{\lambda,i\alpha} s_{\mu,i\alpha}}{m_i}$$

**Jacobian transformation matrix**

$$\hat{P}_{iF} = \sum_{\lambda=1}^{3N} s_{\lambda,iF} \Pi_{\lambda}$$

$$\sum_{i=1}^N \sum_{F=X,Y,Z} \frac{\partial \xi_{\lambda}}{\partial R_{iF}} \frac{\partial R_{iF}}{\partial \xi_{\mu}} = \mathbf{s}_{\lambda,i} \cdot \mathbf{t}_{i,\mu} = \delta_{\lambda,\mu}$$

$$\mathbf{s}_{\lambda,i} = (\mathbf{t}_{i,\mu})^{-1}$$

We solve these linear equations numerically by expanding as polynomials in the small amplitude coordinates.

## General strategy for calculating the eigenvalues and eigenfunctions of the resulting Hamiltonian

*Variational calculation* = Construction of Hamiltonian matrix in terms of suitable basis set, followed by numerical diagonalization of the matrix

A vibrational basis function is generated as a product of „one-dimensional“ (1D) vibrational basis functions, each one describing a single vibrational degree of freedom

A rotation-vibration basis function is obtained by multiplying a vibrational basis function by a symmetric-top eigenfunction.

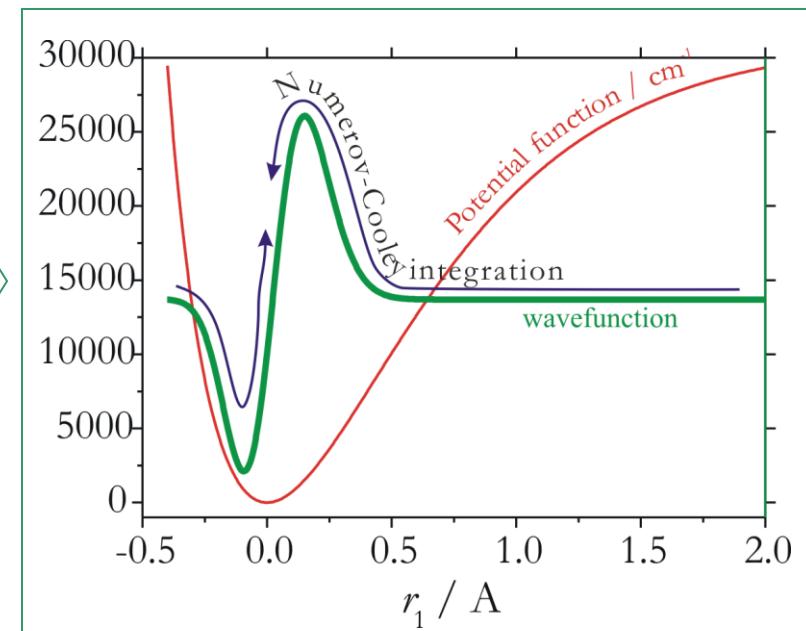
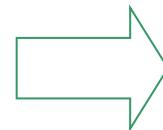
# 1D vibrational basis functions

☞ Harmonic oscillator functions

☞ Morse oscillator functions

☞ Numerical function generated by Numerov-Cooley integration

$$H_{\text{str.}}^{\text{1D}} \psi_{i_s}^{\text{1D}} = E_{i_s}^{\text{str.}} \psi_{i_s}^{\text{1D}}$$

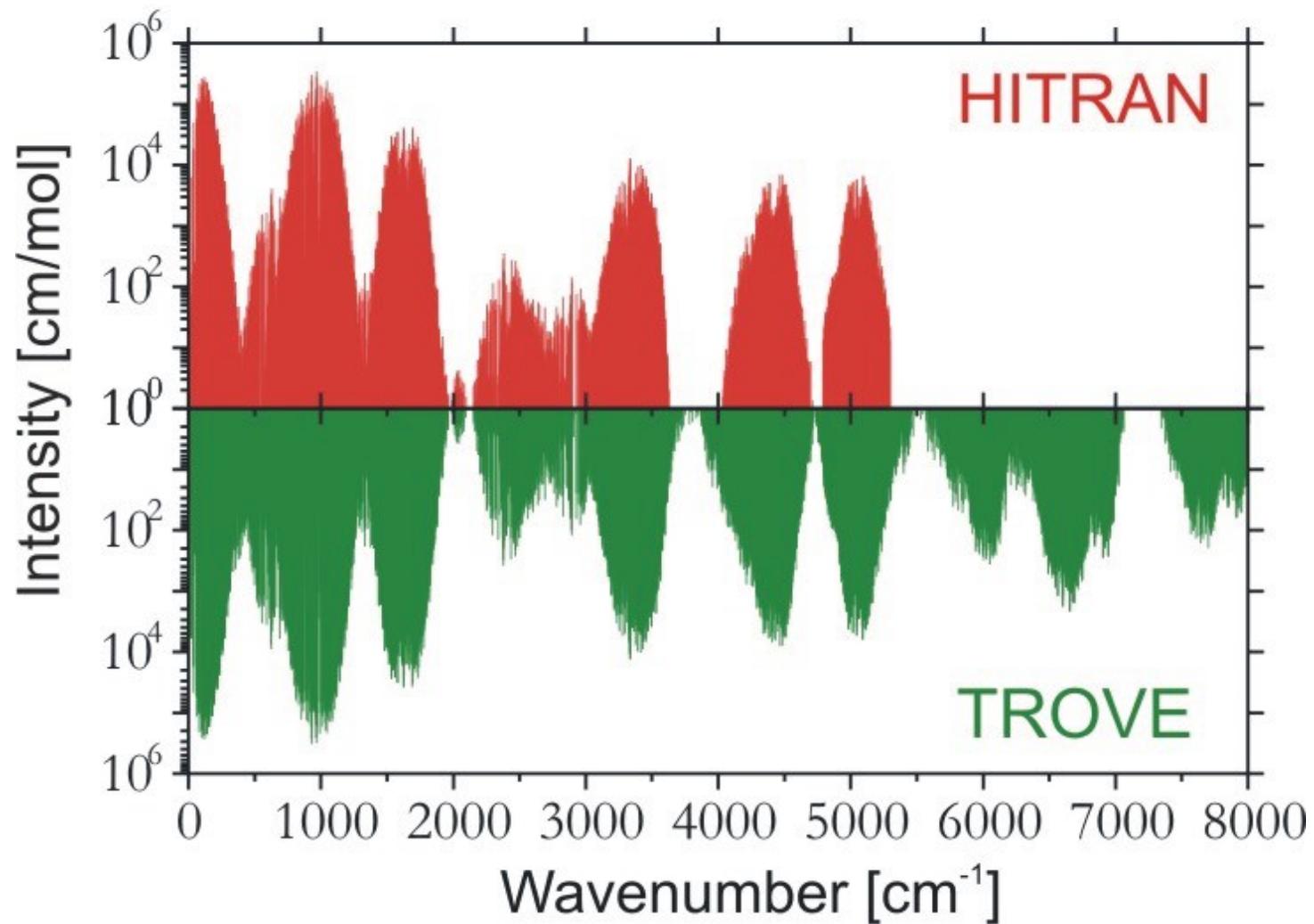


## Rotation-vibration basis functions

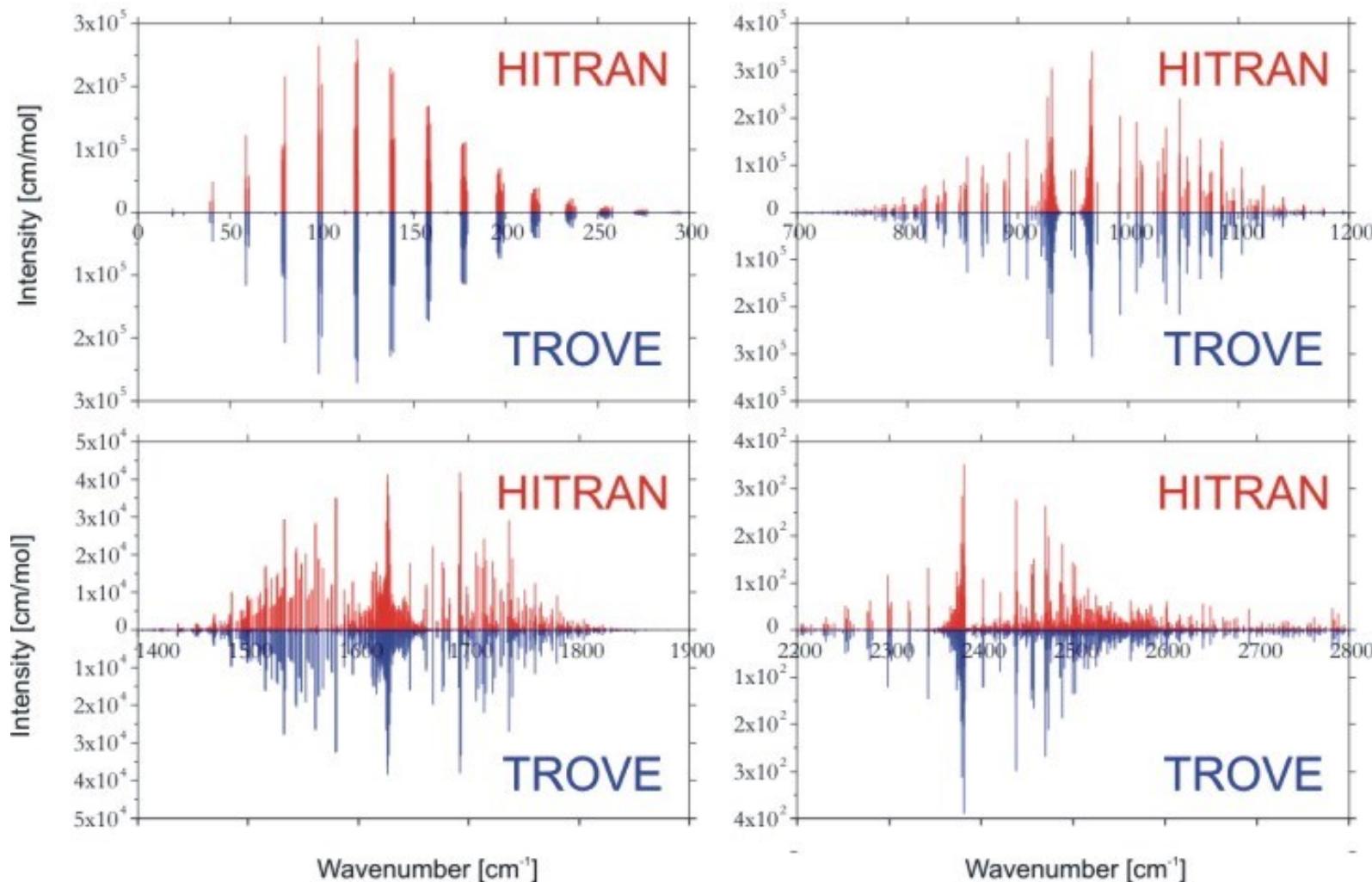
- ☞ Vibrational basis function  $\phi_{\text{vib}} = |n_1\rangle |n_2\rangle |n_3\rangle |n_4\rangle \dots$
- ☞ Rotation-vibration basis function  $\phi_{\text{rot-vib}} = \phi_{\text{vib}} \times |J,k,m\rangle$
- ☞ Construction of matrix
- ☞ (Symmetrization)
- ☞ Numerical diagonalization of matrix
- ☞ (Calculation of intensities from rotation-vibration eigenfunctions)

**Start of parenthetic remark about NH<sub>3</sub> spectrum simulations**

## Parenthetic remark: NH<sub>3</sub> spectrum simulations ( $T=300\text{K}$ )



## NH<sub>3</sub>: Absorption intensities at T=300K, 3.25 million transitions



Absorption intensities at  $T=1500\text{K}$ , 1.1 billion transitions, list is essentially finished, collaboration Yurchenko/Tennyson

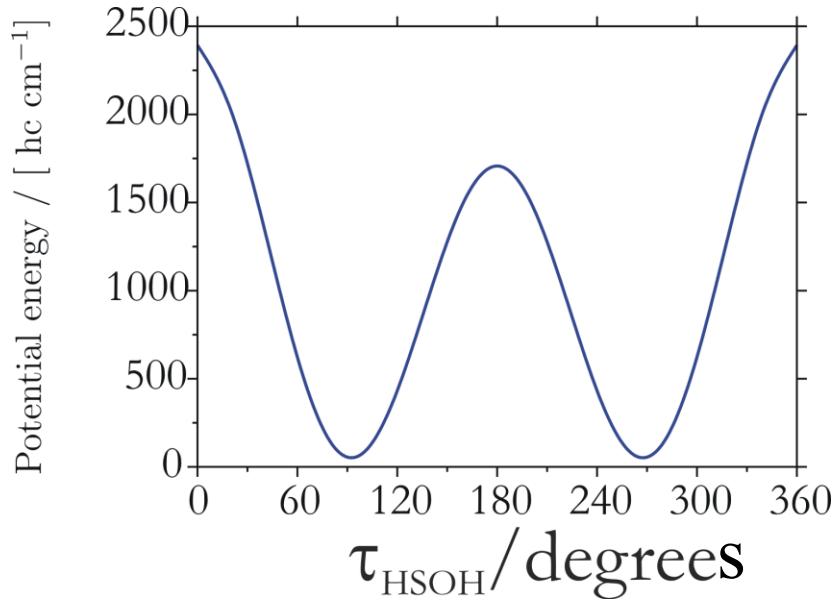
**End of parenthetic remark about  $\text{NH}_3$  spectrum simulations**

**Back to HSOH**

## HSOH: *Ab initio*

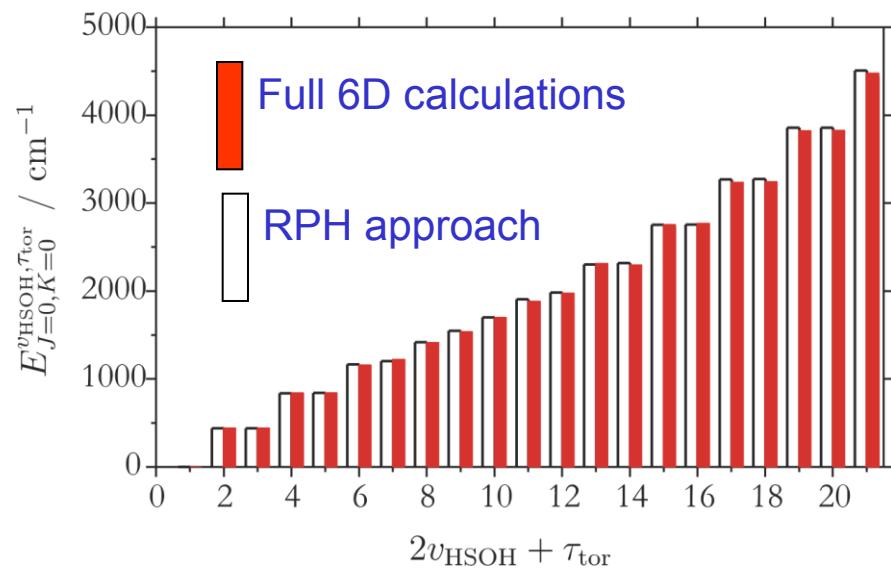
~12000 points

CCSD(T)/aug-cc-pV(Q+d)Z



## TROVE

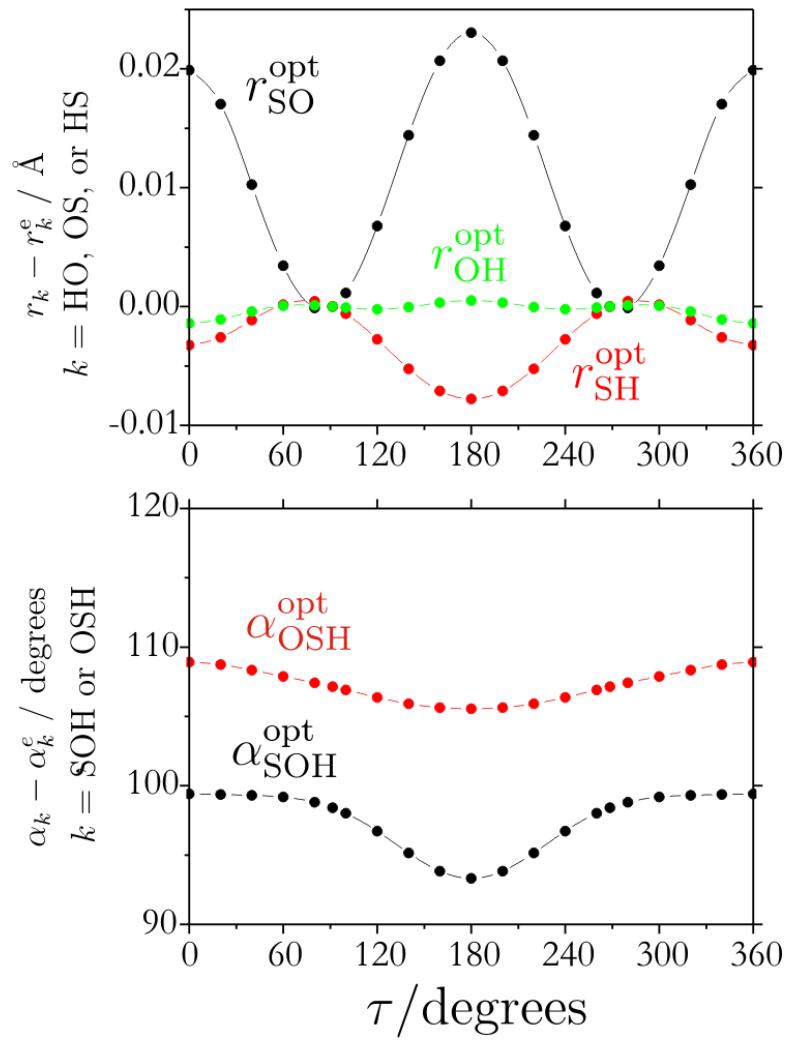
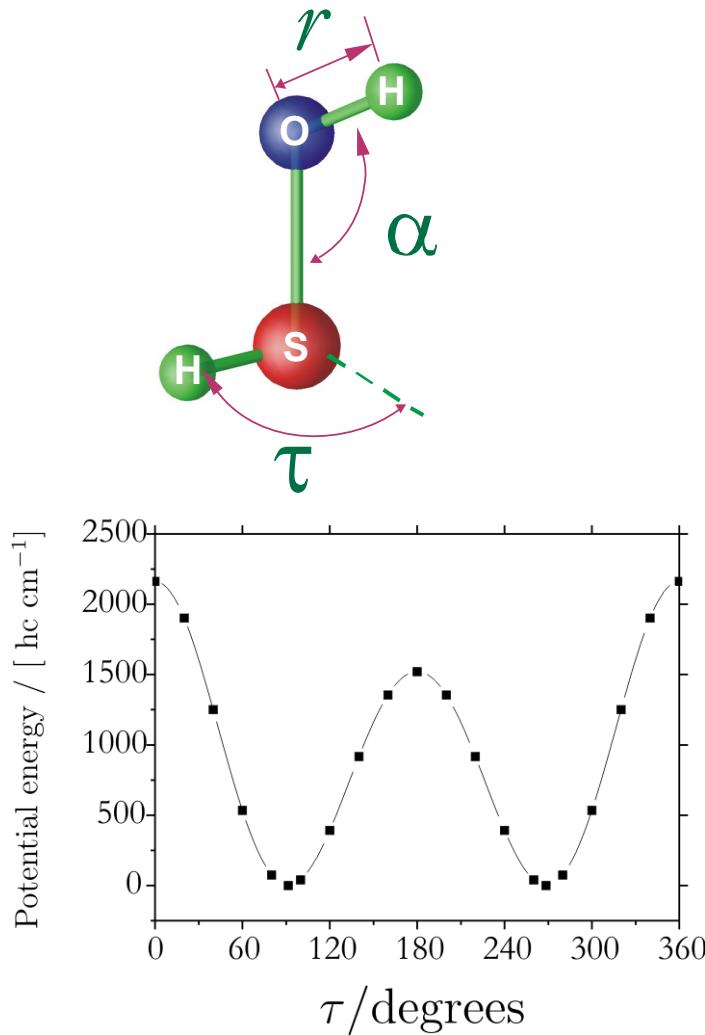
$J = 0 \dots 40, v_{\text{HSOH}} \leq 42$



## RPH approach: expansion around MEP

$$\Psi_{J,\Gamma,i} = |0\rangle |0\rangle |0\rangle |0\rangle |0\rangle |0\rangle \sum_{v_{\text{HSOH}}, \tau_{\text{tor}}, K, \tau_{\text{rot}}} C_{J,\Gamma,i}^{K, \tau_{\text{rot}}, v_{\text{HSOH}}, \tau_{\text{tor}}} |v_{\text{HSOH}}, \tau_{\text{tor}}\rangle |J, K, \tau_{\text{rot}}\rangle$$

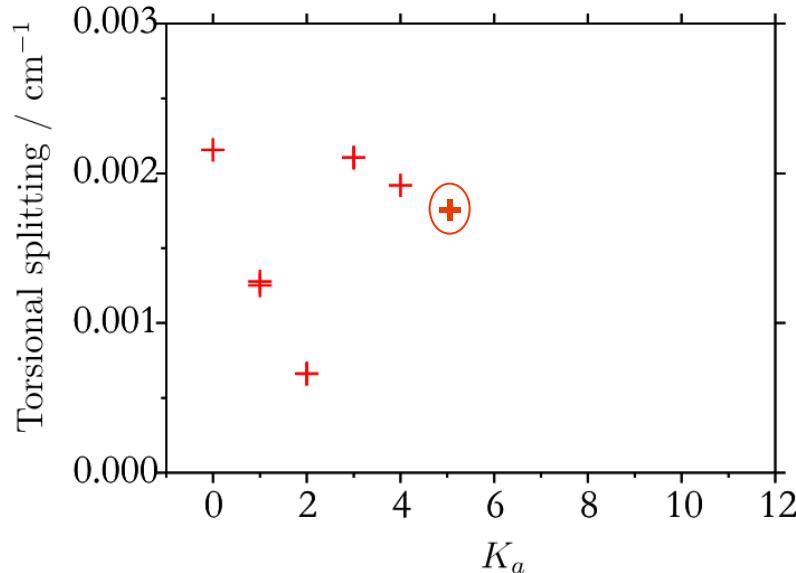
## HSOH: Minimum energy path



## HSOH rotation-torsion levels: experiment vs TROVE

J	$K_a$	$K_c$	$\Gamma$	Term values ( $\text{cm}^{-1}$ )			Splitting ( $\text{cm}^{-1}$ )		
				obs	calc	exp-calc	calc	exp	exp-calc
4	0	4	A'	10.04696	10.05037	-0.00341			
4	0	4	A''	10.04911	10.05252	-0.00341	0.00215	0.00215	0.00000
4	1	3	A''	16.35885	16.36110	-0.00225			
4	1	3	A'	16.36011	16.36236	-0.00225	0.00126	0.00126	0.00000
4	1	4	A'	16.21151	16.21314	-0.00164			
4	1	4	A''	16.21277	16.21440	-0.00163	0.00126	0.00127	0.00000
4	2	2	A''	34.99521	34.99509	0.00011			
4	2	2	A'	34.99698	34.99694	0.00004	0.00184	0.00177	-0.00007
4	2	3	A'	34.99506	34.99495	0.00011			
4	2	3	A''	34.99683	34.99679	0.00004	0.00184	0.00177	-0.00007
4	3	1	A'	66.17016	66.17467	-0.00452			
4	3	1	A''	66.17226	66.17681	-0.00455	0.00213	0.00210	-0.00004
4	3	2	A''	66.17016	66.17467	-0.00452			
4	3	2	A'	66.17226	66.17681	-0.00455	0.00214	0.00210	-0.00004
4	4	0	A'	109.79888	109.82657	-0.02770			
4	4	0	A''	109.80079	109.82854	-0.02774	0.00196	0.00192	-0.00004
4	4	1	A''	109.79888	109.82657	-0.02770			
4	4	1	A'	109.80079	109.82854	-0.02774	0.00196	0.00192	-0.00004

Basis set:  $\nu_{\text{HSOH}} \leq 42$



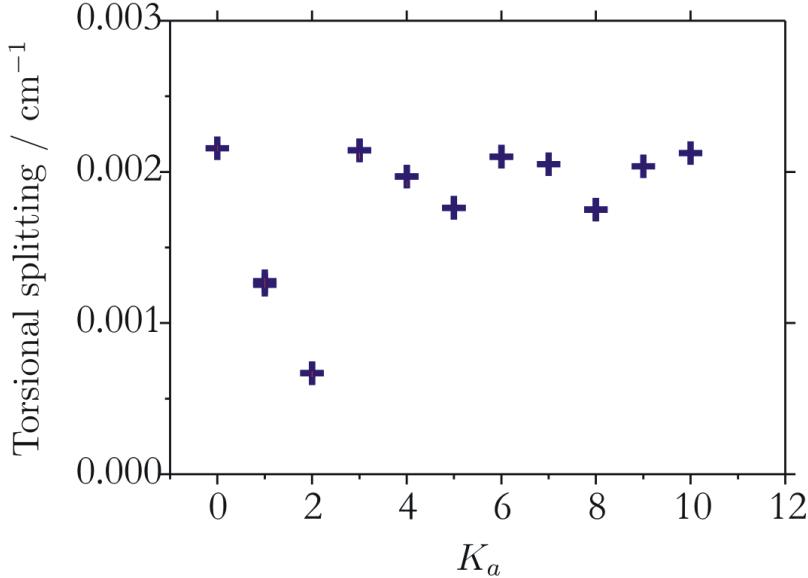
HSOH torsional splittings

Experiment

Cologne Terahertz spectrometer

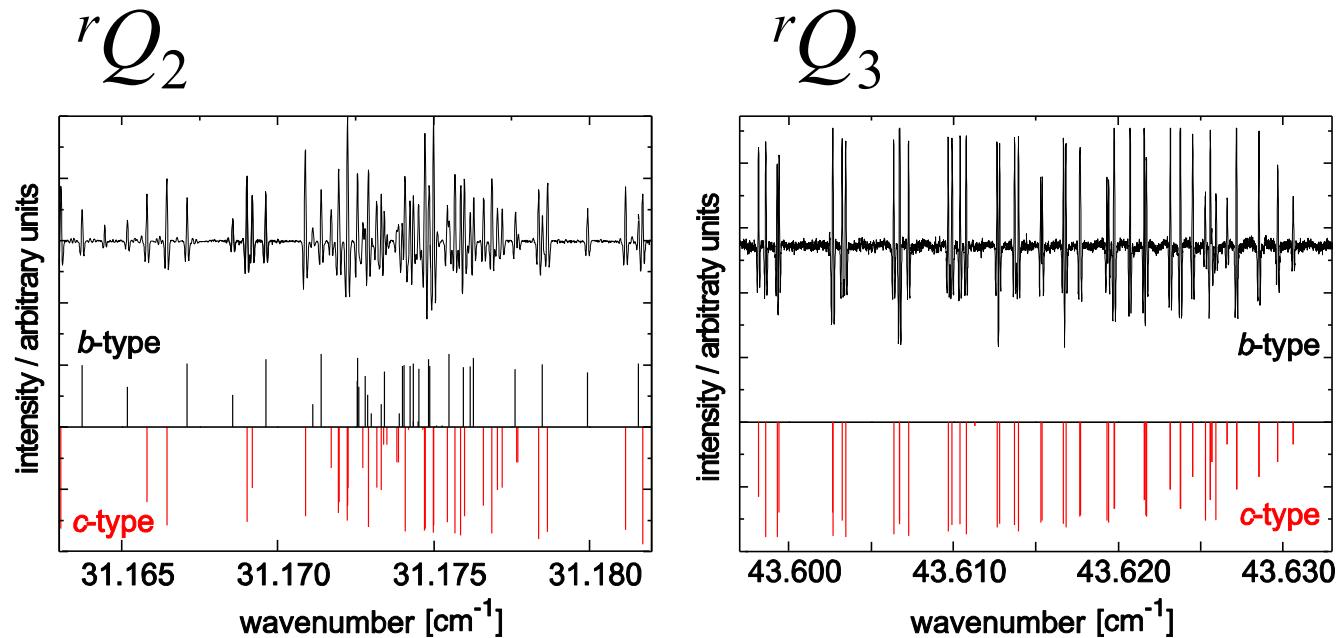
Theory

TROVE calculations

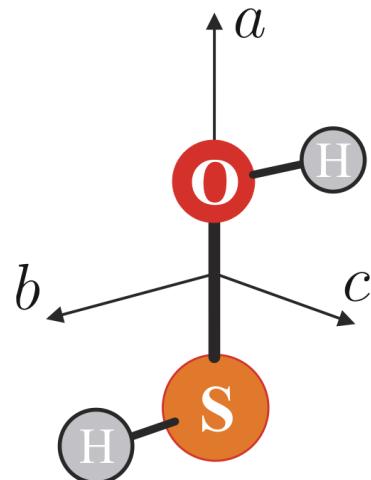


## HSOH “intensity anomaly” in the rotational spectrum

### Experiment:



No *b*-type transitions for  $rQ_3$ .  
Why?



$\bar{\mu}_a, \bar{\mu}_b, \bar{\mu}_c$  are dipole moment components along the principal axes

$\bar{\mu}_a$  is small for HSOH

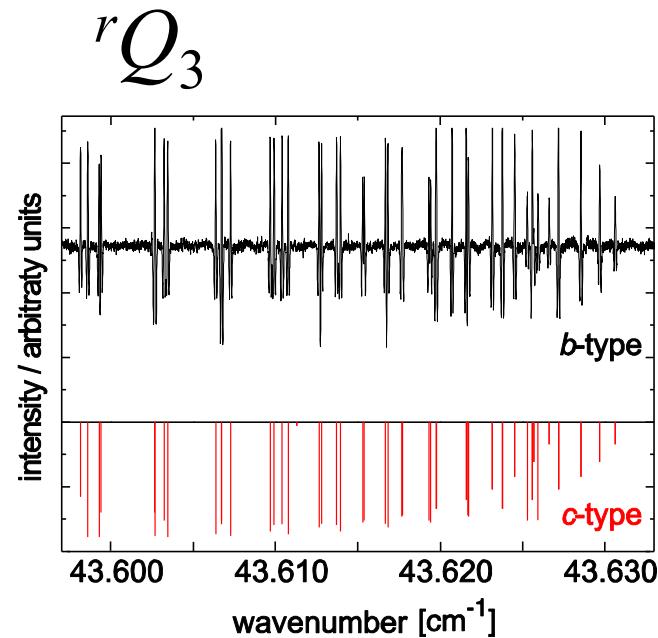
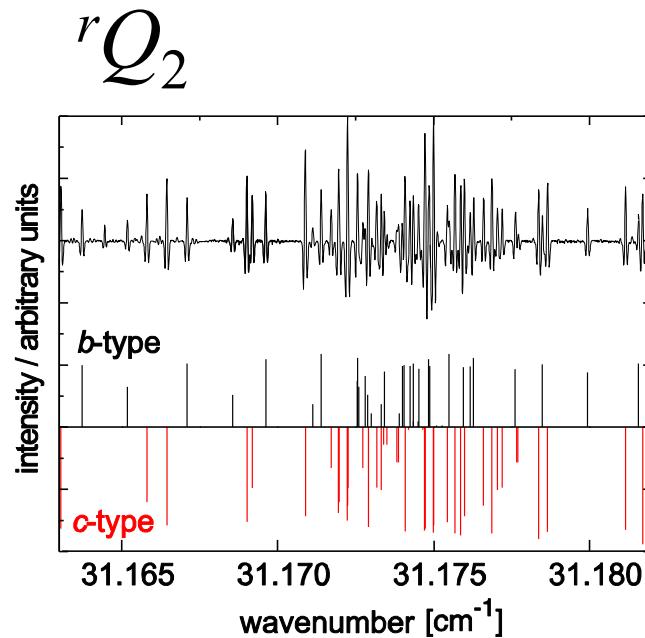
When rotation and vibration are separable, there are *b*-type and *c*-type transitions with the intensity ratio

$$\frac{I_b}{I_c} \approx \frac{\bar{\mu}_b^2}{\bar{\mu}_c^2} \approx 0.28$$

From theoretical dipole moment

**b/c intensity ratio is 0.22 for  $rQ_0$  and 0.23 for  $rQ_1$ .**

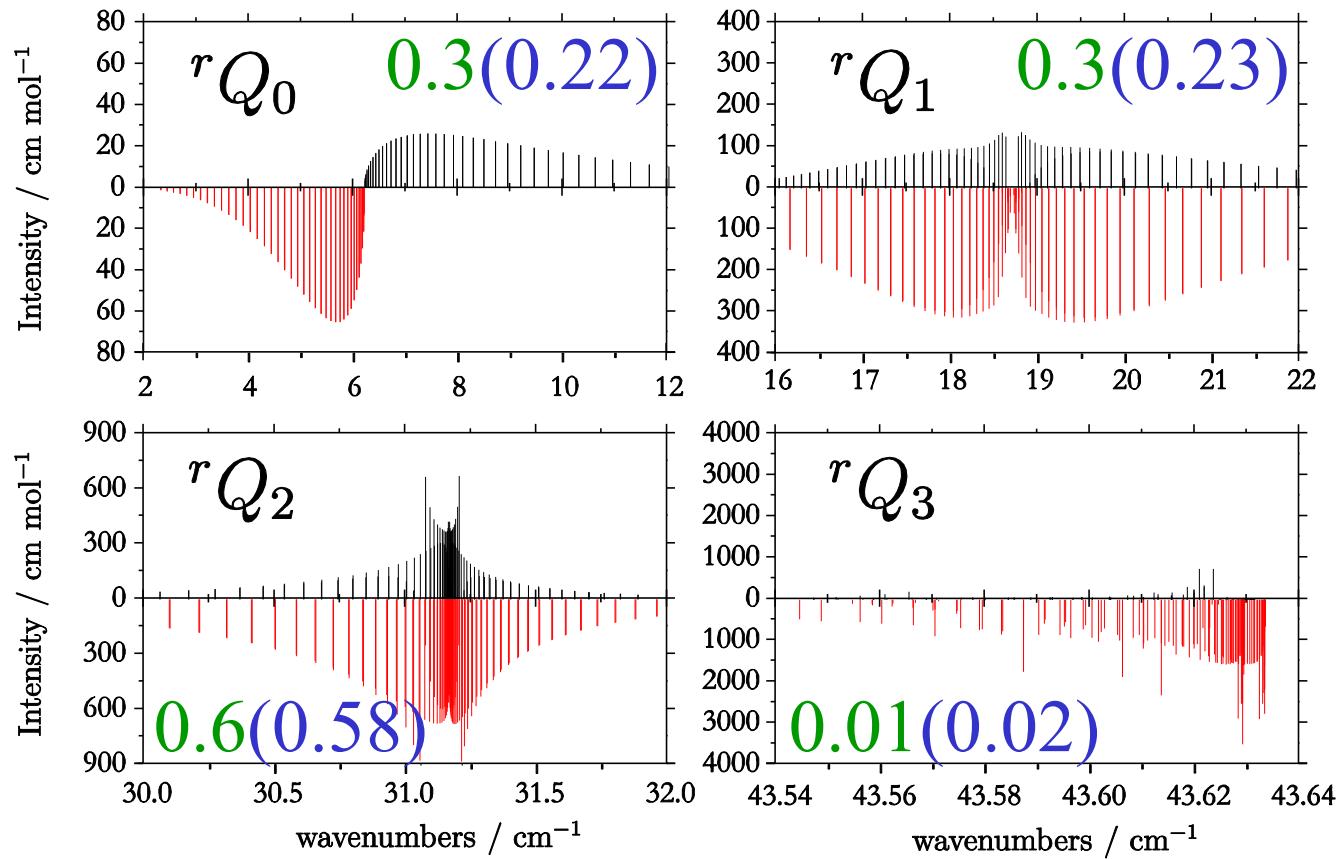
**However, for higher  $K$ :**



$$\frac{I_b}{I_c} \approx 0.58$$

$$\frac{I_b}{I_c} \approx 0.02$$

## TROVE theoretical simulations:



## Explanation: Torsion-rotation interaction

For a given  $K_a$ -value there are four torsion-rotation states  
[two  $K_c$ -values  $\times$  ( $\pm$  torsional parity)]

$$\Psi_{J,K_a,i}^{A'(1)} \approx c_{J,K_a} |J, K_a, 0\rangle \phi_{K_a,0}^{(\text{vib})} + s_{J,K_a} |J, K_a, 1\rangle \phi_{K_a,1}^{(\text{vib})},$$

$$\Psi_{J,K_a,i}^{A'(2)} \approx -s_{J,K_a} |J, K_a, 0\rangle \phi_{K_a,0}^{(\text{vib})} + c_{J,K_a} |J, K_a, 1\rangle \phi_{K_a,1}^{(\text{vib})},$$

$$\Psi_{J,K_a,i}^{A''(1)} \approx c_{J,K_a} |J, K_a, 0\rangle \phi_{K_a,1}^{(\text{vib})} + s_{J,K_a} |J, K_a, 1\rangle \phi_{K_a,0}^{(\text{vib})},$$

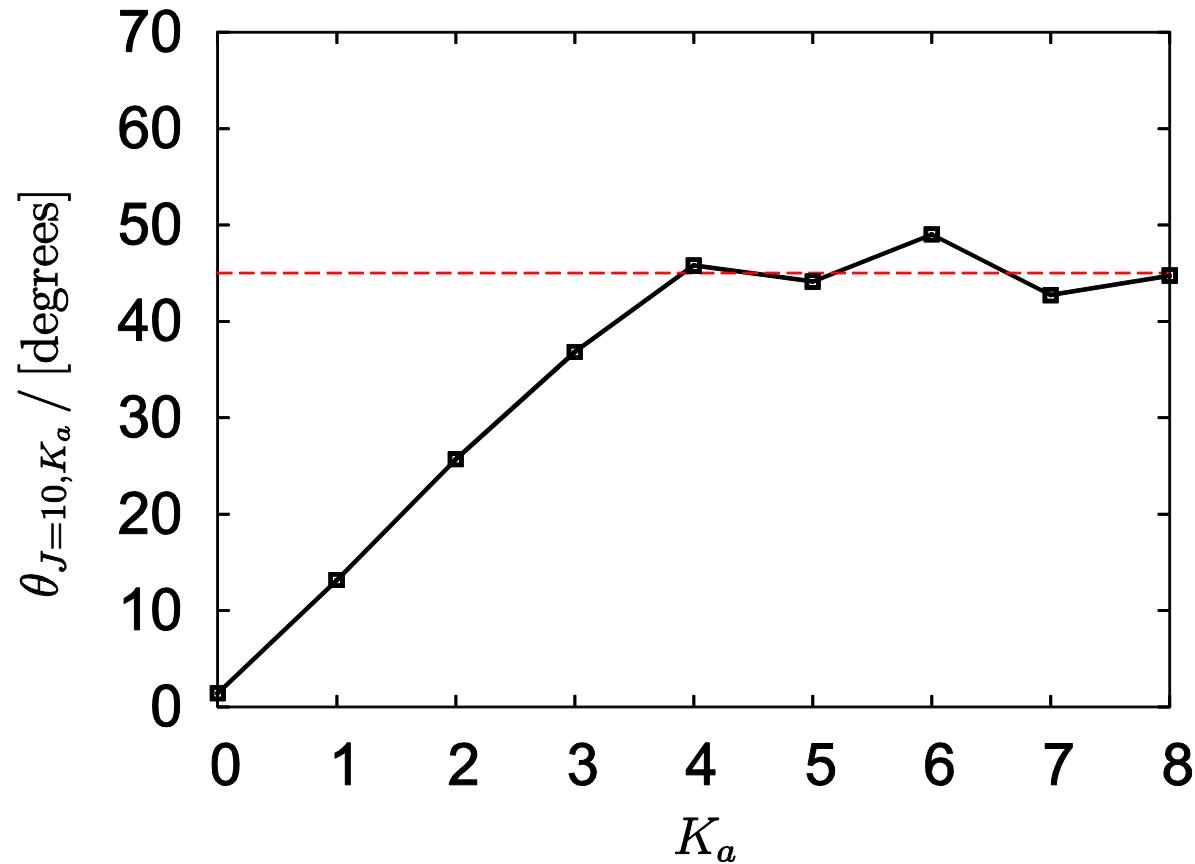
$$\Psi_{J,K_a,i}^{A''(2)} \approx -s_{J,K_a} |J, K_a, 0\rangle \phi_{K_a,1}^{(\text{vib})} + c_{J,K_a} |J, K_a, 1\rangle \phi_{K_a,0}^{(\text{vib})},$$

where  $c_{J,K_a} = \cos(\theta_{J,K_a})$  and  $s_{J,K_a} = \sin(\theta_{J,K_a})$  with the mixing angle  $\theta_{J,K_a} \in [0, \pi/4]$  so that  $c_{J,K_a} \geq s_{J,K_a}$ .

## „Interaction angle“ $\theta_{J,K_a}$

$\theta_{J,K_a} \approx 0^\circ$  for low  $K_a$  – no interaction.

$\theta_{J,K_a} \approx 45^\circ$  for high  $K_a$  – 50-50 mixing.



## Approximate line strengths

$$\begin{aligned}
 S_{A'(1) \leftrightarrow A''(1)} &= S_{A'(2) \leftrightarrow A''(2)} = g_{\text{ns}} A(J, K_a) \\
 &\times \left[ \cos^2 (\theta_{J, K_a+1} - \theta_{J, K_a}) \langle \phi_{K_a, 0}^{(\text{vib})} | \bar{\mu}_b | \phi_{K_a+1, 0}^{(\text{vib})} \rangle^2 \right. \\
 &\quad \left. + \sin^2 (\theta_{J, K_a+1} + \theta_{J, K_a}) \langle \phi_{K_a, 0}^{(\text{vib})} | \bar{\mu}_c | \phi_{K_a+1, 1}^{(\text{vib})} \rangle^2 \right]
 \end{aligned}$$

$$\begin{aligned}
 S_{A'(1) \leftrightarrow A''(2)} &= S_{A'(2) \leftrightarrow A''(1)} = g_{\text{ns}} A(J, K_a) \\
 &\times \left[ \cos^2 (\theta_{J, K_a+1} + \theta_{J, K_a}) \langle \phi_{K_a, 0}^{(\text{vib})} | \bar{\mu}_c | \phi_{K_a+1, 1}^{(\text{vib})} \rangle^2 \right. \\
 &\quad \left. + \sin^2 (\theta_{J, K_a+1} - \theta_{J, K_a}) \langle \phi_{K_a, 0}^{(\text{vib})} | \bar{\mu}_b | \phi_{K_a+1, 0}^{(\text{vib})} \rangle^2 \right]
 \end{aligned}$$

## Approximate line strengths

$$\theta_{J,K_a} \approx \theta_{J,K_a+1} \approx 0^\circ, \text{ low } K_a$$

$$S_{A'(1) \leftrightarrow A''(1)} = S_{A'(2) \leftrightarrow A''(2)} = g_{\text{ns}} A(J, K_a)$$

**b-type**

$$\times \left[ \begin{matrix} \cos^2(\theta_{J,K_a+1} - \theta_{J,K_a}) & \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_b | \phi_{K_a+1,0}^{(\text{vib})} \rangle^2 \\ 1 & \\ + \sin^2(\theta_{J,K_a+1} + \theta_{J,K_a}) & \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_c | \phi_{K_a+1,1}^{(\text{vib})} \rangle^2 \\ 0 & \end{matrix} \right]$$

$$S_{A'(1) \leftrightarrow A''(2)} = S_{A'(2) \leftrightarrow A''(1)} = g_{\text{ns}} A(J, K_a)$$

**c-type**

$$\times \left[ \begin{matrix} \cos^2(\theta_{J,K_a+1} + \theta_{J,K_a}) & \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_c | \phi_{K_a+1,1}^{(\text{vib})} \rangle^2 \\ 1 & \\ + \sin^2(\theta_{J,K_a+1} - \theta_{J,K_a}) & \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_b | \phi_{K_a+1,0}^{(\text{vib})} \rangle^2 \\ 0 & \end{matrix} \right]$$

## Approximate line strengths

$$\theta_{J,K_a} \approx \theta_{J,K_a+1} \approx 45^\circ, \text{ high } K_a$$

$$S_{A'(1) \leftrightarrow A''(1)} = S_{A'(2) \leftrightarrow A''(2)} = g_{\text{ns}} A(J, K_a)$$

„Hybrid“

$$\times \left[ \begin{array}{l} \cos^2 (\theta_{J,K_a+1} - \theta_{J,K_a}) \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_b | \phi_{K_a+1,0}^{(\text{vib})} \rangle^2 \\ + \sin^2 (\theta_{J,K_a+1} + \theta_{J,K_a}) \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_c | \phi_{K_a+1,1}^{(\text{vib})} \rangle^2 \end{array} \right]$$

1                                    1

$$S_{A'(1) \leftrightarrow A''(2)} = S_{A'(2) \leftrightarrow A''(1)} = g_{\text{ns}} A(J, K_a)$$

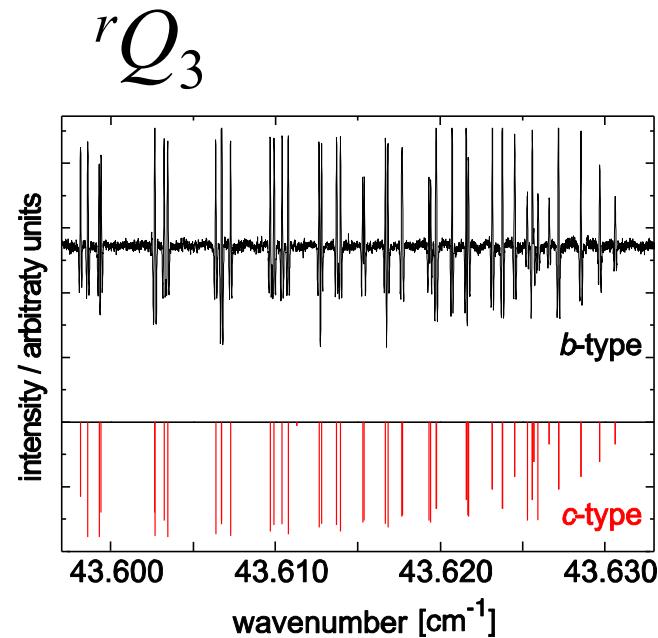
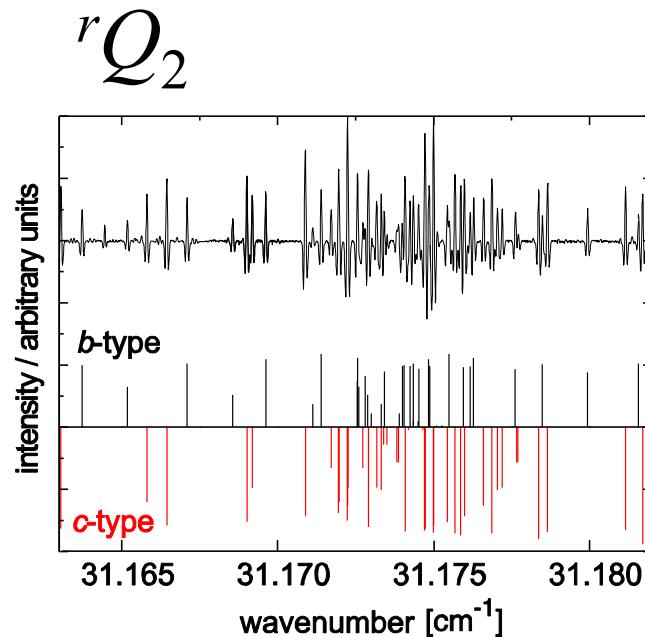
Missing

$$\times \left[ \begin{array}{l} \cos^2 (\theta_{J,K_a+1} + \theta_{J,K_a}) \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_c | \phi_{K_a+1,1}^{(\text{vib})} \rangle^2 \\ + \sin^2 (\theta_{J,K_a+1} - \theta_{J,K_a}) \langle \phi_{K_a,0}^{(\text{vib})} | \bar{\mu}_b | \phi_{K_a+1,0}^{(\text{vib})} \rangle^2 \end{array} \right]$$

0                                    0

**b/c intensity ratio is 0.22 for  $rQ_0$  and 0.23 for  $rQ_1$ .**

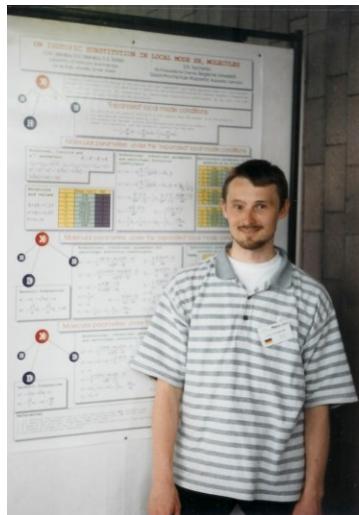
**However, for higher  $K$ :**



$$\frac{I_b}{I_c} \approx 0.58$$

$$\frac{I_b}{I_c} \approx 0.02$$

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