Name: **LZ - number:**

Protocol for experiment no. 55

Determination of measurement errors

To be completed !! Obligation to return !! additional 1 graph on graph paper
Latest date of return: 1st PC-experiment

I. Aim of the experiment:

The following experiment is intended to verify Boyle-Mariotte's law for air both graphically and mathematically. Computationally, this is done by curve fitting using linear regression; the results of both evaluation methods are to be compared.

II. Fundamentals:

An ideal gas has the pressure p and the volume v. Then, according to Boyle and Mariotte, the following applies for all changes of state at constant temperature

(1) $p \bullet v = \text{const.}$

This means: If you halve the volume of a gas, its pressure doubles.

Equation (1) is also known as the isothermal equation of state of ideal gases. It represents a special case of the ideal gas law

(2) $p \bullet v = n \bullet R \bullet T$, with n = number of moles and R = general gas constant

Mathematically, equation (1) corresponds to the function of a hyperbola in a p, v diagram. This function can be described as a straight line by simple transformation,

(3) $1/p = \text{const.} \bullet v$

The transformation to a straight line offers the practical advantage of a simple check of the postulated relationship between p and v, which can be carried out both graphically and mathematically.

Graphically, the “best” straight line is the one to which the most points fit “by eye”. Mathematically, on the other hand, it is the one for which the sum of the squares of the deviations of the individual measuring points from this straight line is a minimum.

In contrast to the first method, this method treats all points objectively the same. Is the result of an experiment n pairs of values x_i / y_i whose functional relationship is given by a straight line ($y = m x + b$), then the “method of least squares” gives the following result:

$$(4) \qquad m = \frac{n * \sum_i x_i y_i - \sum_i x_i * \sum_i y_i}{n * \sum_i x_i^2 - (\sum_i x_i)^2}$$

$$(5) \qquad b = \frac{\sum_i y_i * \sum_i x_i^2 - \sum_i x_i * \sum_i x_i y_i}{n * \sum_i x_i^2 - (\sum_i x_i)^2}$$

The correlation coefficient r is used as a measure of the quality of the fit, which is defined by

$$(6) \qquad r = \frac{(n * \sum_i x_i y_i - \sum_i x_i * \sum_i y_i)^2}{(n * \sum_i x_i^2 - (\sum_i x_i)^2) * (n * \sum_i y_i^2 - (\sum_i y_i)^2)}$$

r = 0 : no correlation

r = 1 : complete correlation $\qquad 0 \leq r \leq 1$!!!!

The better the correlation is fulfilled, the closer r is to 1.

III. Experimental setup and execution:

The experiment was carried out with a volumometer, as shown in Fig. 1.

It consists of a glass vessel that can be closed at the top with a tap, after which it contains a gas (air) with the initial volume v_0 and pressure p_0.
It runs out at the bottom into a 25 cm long, calibrated, open tube. This tube has a cross-section of ($q = 0.587\ cm^2$), so that the respective change in volume Δv can be calculated by measuring the length l_1 when it is lifted out of the glass vessel filled with mercury (Fig. 2). With the stopcock open, the volumometer was initially immersed in the mercury until the zero mark on the scale coincided with the top of the mercury meniscus inside and outside the tube.

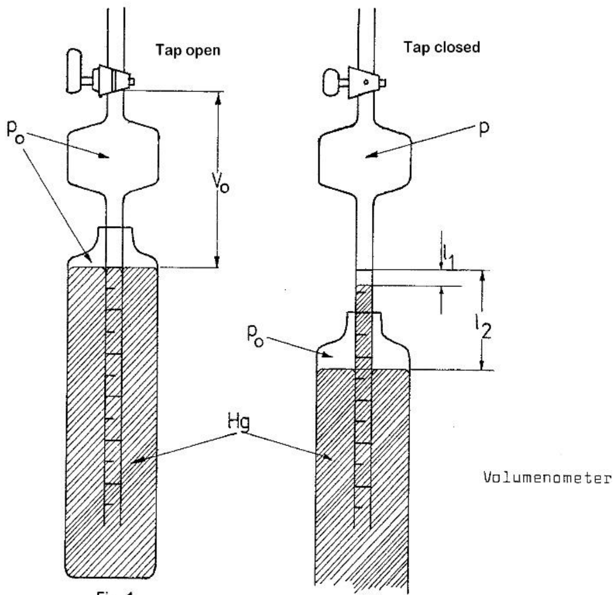

Fig. 1 Fig. 2

p_0 is the external air pressure prevailing at the time of the experiment and could be determined using a barometer.
The volumometer was lifted out in 10 steps of about 20 mm with the stopcock closed.
The values for l_1 and l_2 are summarized in Table 1.

The pressures p prevailing in the volumometer for the individual measurements can now be obtained from equation

(7) $p = p_0 - \Delta p$, where $\Delta p = l_2 - l_1$ and the corresponding volumes v from

(8) $v = v_0 + \Delta v$.

If Boyle-Mariotte's law applies, the following must apply:

(9) $p \bullet v = p \bullet (v_0 + \Delta v) = p_0 \bullet v_0$ and by transforming into a straight line:

(10) $$\frac{1}{p} = \frac{1}{p_0 \bullet v_0} \bullet \Delta v + \frac{1}{p_0}$$ (corresponds to $y = m \bullet x + b$)

Task:

Make the experiment clear to you. Transfer the measured values (obtained on the extra sheet of paper) to the following Table 1 and complete the evaluation of these measured data yourself!

Air pressure:	p_0= [mm]					
	q = 0.587 cm^2 = 58.7 mm^2					
Measurement	values					
Nr.	**l_1 [mm]**	**l_2 [mm]**	$\Delta p = l_2 - l_1$ [mm]	**Δv** = $l_1 * q$ $[mm]^3$	$p = p_0 - \Delta p$ [mm]	**1/p** $[mm]^{-1}$

Tab. 1: Table of measured values

a) Plot 1/p in a diagram as a function of Δv (graph paper!!)
Determine the gradient and intercept of the resulting straight line graphically.
Determine v_0 and p_0!!!
P_0 should correspond to the initial pressure (on your data sheet) in terms of size

b) Create a table of values for the required values of Eq. 4 and Eq. 5 (see Tab. 2 below) and calculate the gradient and intercept using linear regression (equations 4-6).

Determine v_0, p_0 from the slope and intercept of this regression line and compare the values with those of the graphical evaluation (a) !!!

What is the correlation coefficient?

	X_i	Y_i	X_i^2	Y_i^2	X_iY_i
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
Σ					
Σ^2					

Table 2: Table of measured values for the linear regression $X = \Delta v$ and $Y = 1/p$

Results:

graphical a)	gradient m [mm^{-4}]:		v_0=	p_0=
	b in [mm^{-1}]:			

regression b)	gradient m [mm^{-4}]:		v_0=	p_0=
	b in [mm^{-1}]:			
	r =			

Appendix: Diagram 1/p against Δv (graph paper)