

Experiment No. 58

Dynamic viscosity of gases (Hagen-Poiseuille's law)

Keywords:

Kinetic gas theory, ideal gas, characteristic quantities for the description of gases (see below), Hagen-Poiseuille's law, laminar flow, dynamic viscosity

Literature:

Atkins, "Physical Chemistry"

Barrow, "Physical Chemistry", Volume 1

Gerthsen, Kneser, Vogel "Physics"

Kohlrausch, "Practical Physics", Volume 1

Fundamentals:

The kinetic theory of gases explains the macroscopic behavior of gases, in particular their thermal properties, from the motion of the gas particles. Although the movement of an individual particle cannot be tracked, it is still possible to make accurate statistical statements about the whole, due to the very large number of particles (1 mole = 6.023×10^{23} particles) and their relatively high speed. To this end, the following simplifications are made, among others, which apply strictly only to the ideal gas, but are also sufficiently well fulfilled for many real gases at not too high pressures and not too low temperatures:

- 1) The particles are so small compared to their distances that they can be treated as dimensionless mass points.
- 2) The particles do not exert any mutual forces on each other, except in the case of collisions.
- 3) The conservation laws (momentum, energy) of classical mechanics apply to the collision of two particles and to the collision of a particle with the wall of a vessel.
- 4) At temperature equilibrium, the motion of the particles is completely random, i.e. no spatial direction is favored.
- 5) All particles have a temperature-dependent velocity distribution (Maxwell-Boltzmann).

These general basic assumptions can be used to derive characteristic quantities to describe the behavior of the gas. Some of these are:

- the pressure: (1)
$$p = \frac{1}{3} * N * m * \bar{u}^2$$

where N = number of particles per unit volume

m = mass of particles

$\bar{u}^2$ = mean of the squares of all velocities

According to the ideal gas law: (1a)
$$p = N * k * T$$

- Characteristic velocities of gas particles:

most frequent velocity: (2a)
$$u_h = \sqrt{2 * kT / m}$$

average velocity: (2b)
$$\bar{u} = \sqrt{\frac{8}{\pi} * kT / m}$$

Root of the averaged velocity squares: (2c)
$$\sqrt{\bar{u}^2} = \sqrt{3 * kT / m}$$

- the dynamic viscosity: (3)
$$\eta = \frac{1}{3} * N * m * \bar{u} * \lambda$$

- the mean free path: (4)
$$\lambda = \frac{1}{\sqrt{2} * \pi * d^2 * N}$$

where d = particle diameter and $\pi * d^2$ = impact cross-section for a particle

- the number of collisions of one particle with all others per second

(5)
$$Z_1 = \sqrt{2} * \pi * d^2 * \bar{u} * N$$

- the number of all collisions per second and cm³:

(6)
$$Z_{11} = \frac{1}{2} * N * Z_1 \text{ (why factor } \frac{1}{2} \text{ ?)}$$

When one layer of gas moves along another, the particles that fly from one layer into the second through the imaginary separating surface (diffusion) cause the two to interlock to a certain extent. This tends to slow down the faster layer and accelerate the slower one. The

quantity “viscosity” (η) is introduced as a measure of the force that influences the mutual displacement. In the SI system, the unit of viscosity is the Pascal second ($\text{Pa}\cdot\text{s}$):

$$(7) \quad 1 \text{ Pa} \cdot \text{s} = 1 \text{ N} \cdot \text{s} \cdot \text{m}^{-2} = 1 \text{ kg} \cdot \text{s}^{-1} \cdot \text{m}^{-1}$$

The older unit Poise (P, from Poiseuille) has the following relationship:

$$(8) \quad 1 \text{ P} = 0.1 \text{ Pa} \cdot \text{s} = 1 \text{ g} \cdot \text{s}^{-1} \cdot \text{cm}^{-1}$$

Here, 1 Pascalsecond is equal to the dynamic viscosity of a laminar flowing homogeneous fluid (liquid, gas) in which the shear stress 1 Pa prevails between two layers arranged in parallel at a distance of 1 m with a velocity difference of $1 \text{ m} \cdot \text{s}^{-1}$.

Under normal conditions (0°C , 1 atm), the viscosity of many gases is in the order of $10^{-5} \text{ Pa}\cdot\text{s}$, that of liquids between 1 and $10^{-4} \text{ Pa}\cdot\text{s}$.

In contrast to liquids, the viscosity of the ideal gas increases with increasing temperature ($\eta \propto \sqrt{T}$). It is **not dependent on pressure**, as the effect of the change in density with pressure is compensated by the change in mean free path that occurs.

The viscosity of real gases shows above all a stronger dependence on temperature and at higher gas densities also a dependence on pressure. This can be explained by larger intermolecular interactions and a more complicated form of momentum transport than in the ideal gas.

For a laminar stationary flow through a capillary, the following applies according to Hagen and Poiseuille (for derivation see Gerthsen):

$$(9) \quad V = \frac{\pi \cdot \Delta p \cdot r^4 \cdot t}{8 \cdot l \cdot \eta} \quad \text{where } T = \text{const.}$$

Here V is the volume of fluid flowing through a capillary tube of radius r and length l in time t under the effect of the pressure difference Δp .

Eq. (9) assumes that the entire pressure energy is converted into heat by friction and that the flow in each cross-section has a parabolic Poiseuille velocity distribution over its entire length. Strictly speaking, equation (9) only applies to liquids; for gases, the expansion due to the pressure drop and the sliding along the wall must also be taken into account.

However, these corrections are generally so small that Eq. (9) is a useful approximation. However, if the mean free path of the gas particles reaches the order of magnitude of the vessel dimension, the amount of gas flowing through the capillary will be much greater than calculated by Eq. (9).

Task:

Using a Rankine viscometer (Fig. 1) and a stopwatch, determine the times required for the gases N₂, He and CO₂ to flow through a capillary. Using the known dynamic viscosity of N₂ η (N₂) = $1.748 \cdot 10^{-5}$ Pa*s at 20 °C), gas kinetic quantities are calculated.

The experimental setup:

In the Rankine viscometer, the gas to be tested is forced through a capillary using a drop of mercury in a circuit (Fig. 1). The markings S₁ and S₂ delimit the measuring section.

Procedure:

- 1) Before connecting the gas line to the apparatus, familiarize yourself with the safe handling of the cylinder valves as instructed by the assistant!
Practice several times!
- 2) The drop of mercury is placed in one of the bulges in the pipe extension below (above) S₁ or S₂.

3) The taps H₁ and H₂ are opened. The gas line from the nitrogen cylinder is connected to H₁ (or H₂). The apparatus is flushed with nitrogen for a few minutes.

(Caution! Do not let it flow in if the overpressure is too high!)

Then stop the gas supply and close the two taps.

- 4) Quickly turn the apparatus 180 degrees. Use the stopwatch to measure the falling time of the drop of mercury between the markings S₁ and S₂ (or S₂ and S₁). This procedure is repeated six times.
- 5) For the gases CO₂ and He, the points 2) to 4) are carried out accordingly.

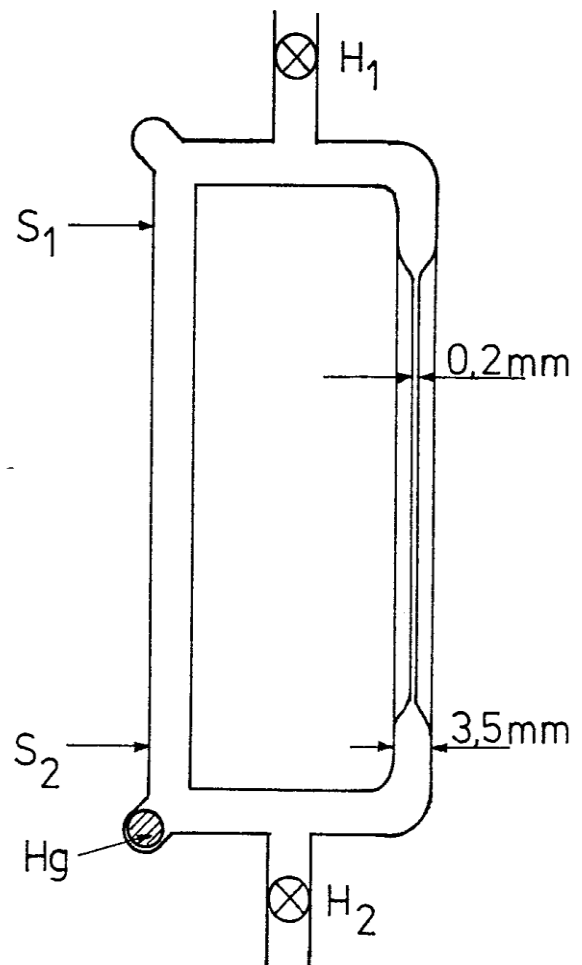


Fig. 1: Schematic diagram of a Rankine viscometer

Evaluation:

- 1) The Hagen-Poiseuille law applies to the fall time of the drop of mercury:

$$(10) \quad t = \frac{8 * V * l * \eta}{\pi * \Delta p * r^4}$$

Since all variables except η and t remain constant due to the construction of the Rankine viscometer, **relative values** of the dynamic viscosity can be calculated very easily by measuring the time.

It is: (11) $t_1 / t_2 = \eta_1 / \eta_2$

- 2) Determine the arithmetic mean value for the falling time of the drop of mercury from your multiple measurements for each gas.
- 3) Using the mean values, calculate the dynamic viscosity of CO₂ and He relative to that of nitrogen.
- 4) Using equations (1) to (6), calculate the characteristic values u , λ , d , Z_1 and Z_{11} for all three gases. Clearly present the results together with the viscosities in a table.
- 5) Discuss the accuracy of your measurement:
- a) What is the relative and absolute error of the values obtained for the dynamic viscosity?
 - b) Obtain data on the temperature dependence of the dynamic viscosity and estimate how large the error will be with a temperature change of 1 °C!

Accessories:

Rankine viscometer, stopwatch, connection line for the gas cylinders.